KATHOLIEKE UNIVERSITEIT LEUVEN

FP7 project "**R**ailway **I**nduced **V**ibrations **A**batement **S**olutions"
WP4 – Vibration mitigation measures on the transmission/propagation path

A numerical study of subgrade stiffening as a mitigation measure for railway induced vibration

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Scope of WP4

- Develop and optimize vibration reduction technologies on the transmission/propagation path
- Mitigation measures should be under or next to the track (to be regarded as part of the railway infrastructure)

Options

- Trenches
- Buried wall barriers
- Subgrade stiffening
- Horizontally layered wave impeding blocks
- Wave reflectors (resonating masses)

Objective

- To gain insight in the underlying physical mechanism
- To support the design of efficient and cost-effective vibration mitigation measures by means of numerical analyses

Subgrade stiffening

- Is often applied for railway tracks on soft soils to improve the soil beneath the track (reducing settlements or track displacements)
- Common techniques:
  - deep vibro compaction
  - vibro replacement
  - deep soil mixing
  - gravel or cement columns
  - hydraulic fracture injection with stable cement-bentonite mixtures
  - vacuum consolidation
  - ...

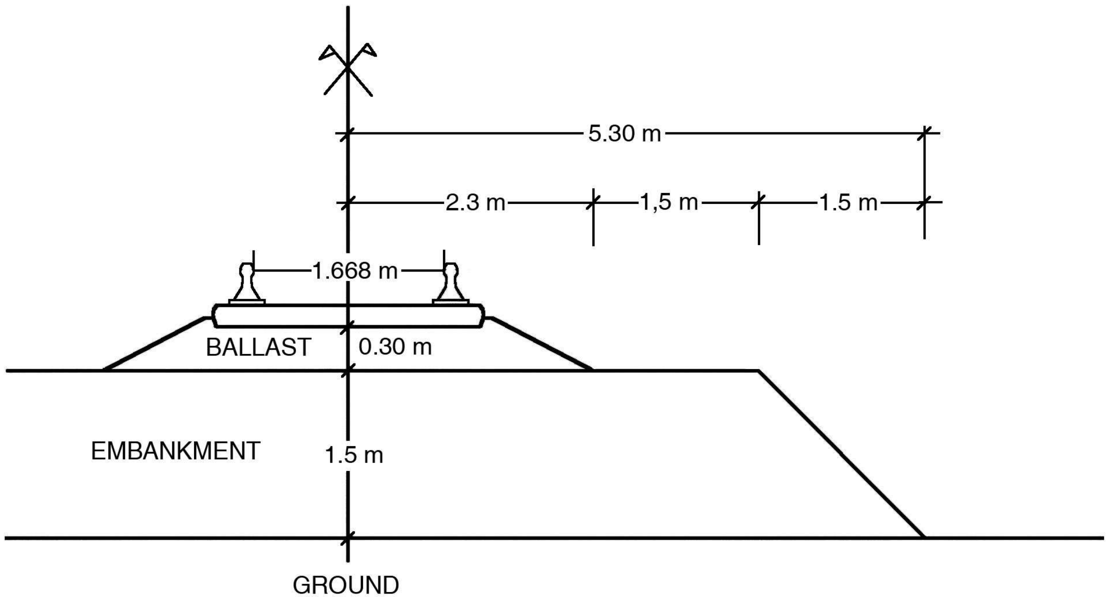
Lay-out

- Stiffening under the track: more efficient, but risk of track uplifting (undesired)
- Stiffening next to the track: wave impeding barrier

Track characteristics (Gerona test site)

- UIC 54 rails: Euler-Bernoulli beams
- Rail pads: continuous spring-damper connections
- Sleepers: uniformly distributed mass, rigid in the plane of the track cross-section
- Ballast: elastic continuum
- Embankment: elastic continuum

RIELLS CROSS-SECTION



Methodology

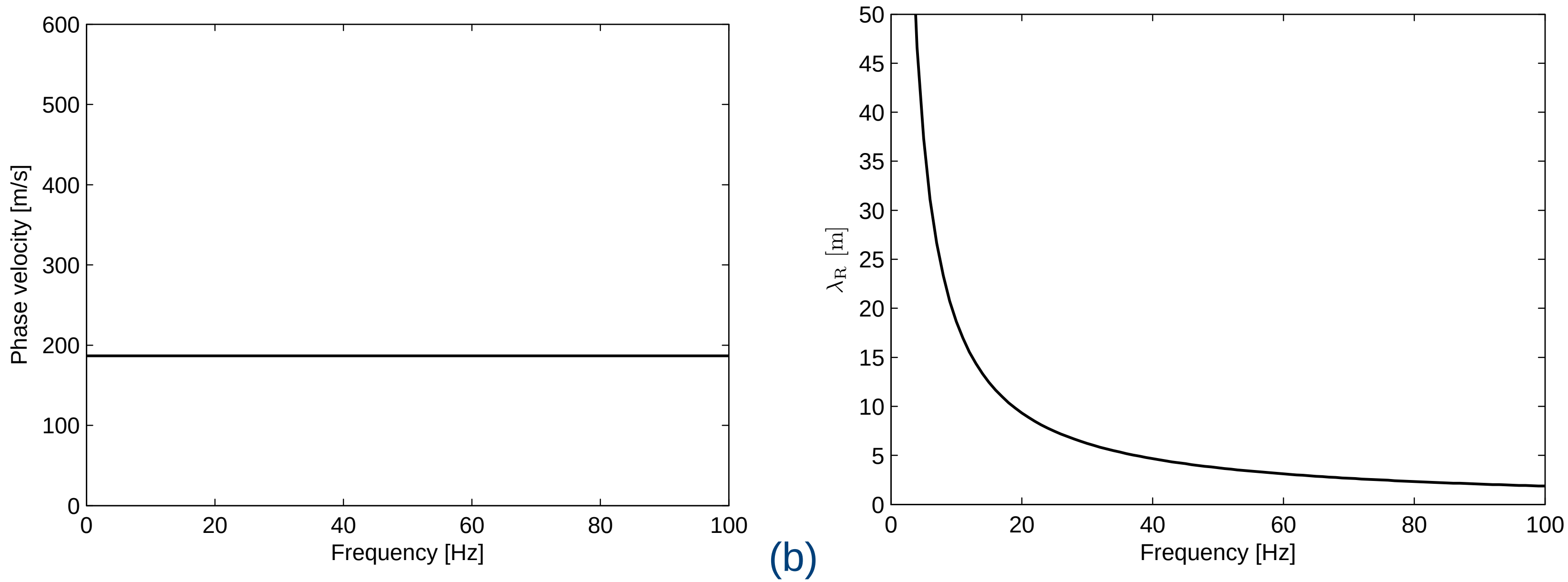
- Coupled FE–BE method
- 2.5D formulation (transformation from y to k_y) [François et al., CMAME, 2010]

Homogeneous visco-elastic halfspace

- Dynamic characteristics of the halfspace

Layer	Thickness $[\mathrm{m}]$	C_s $[\mathrm{m/s}]$	C_p $[\mathrm{m/s}]$	β_s $[-]$	β_p $[-]$	ρ $[\mathrm{kg/m^3}]$
1	∞	200	400	0.025	0.025	2000

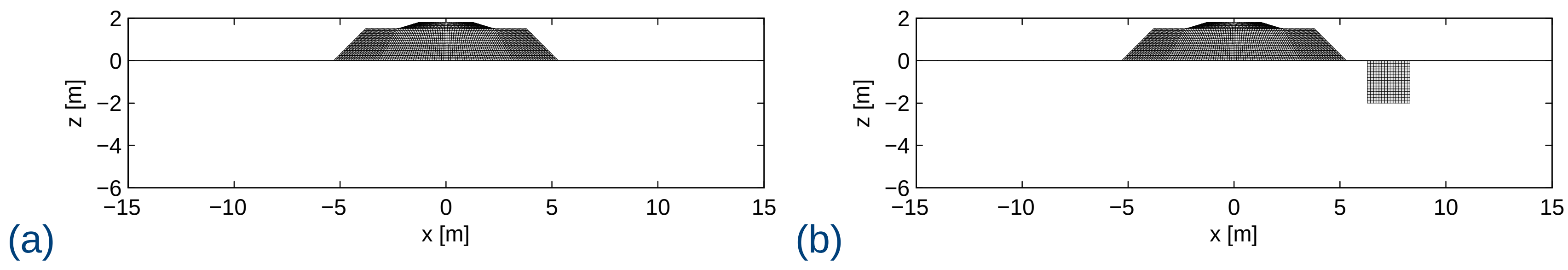
- Non-dispersive behaviour
- Only one mode exists
- (a) Phase velocity C_R and (b) wavelength $\lambda_\mathrm{R} = C_\mathrm{R}/f$ vs. frequency f



(a) (b)

Variant A

- (a) Reference case and (b) stiffening next to the track (variant A)



- Stiffened block of soil
  - Square cross section: $w = 2\,\mathrm{m}$, $h = 2\,\mathrm{m}$
  - Dynamic characteristics (Keller):

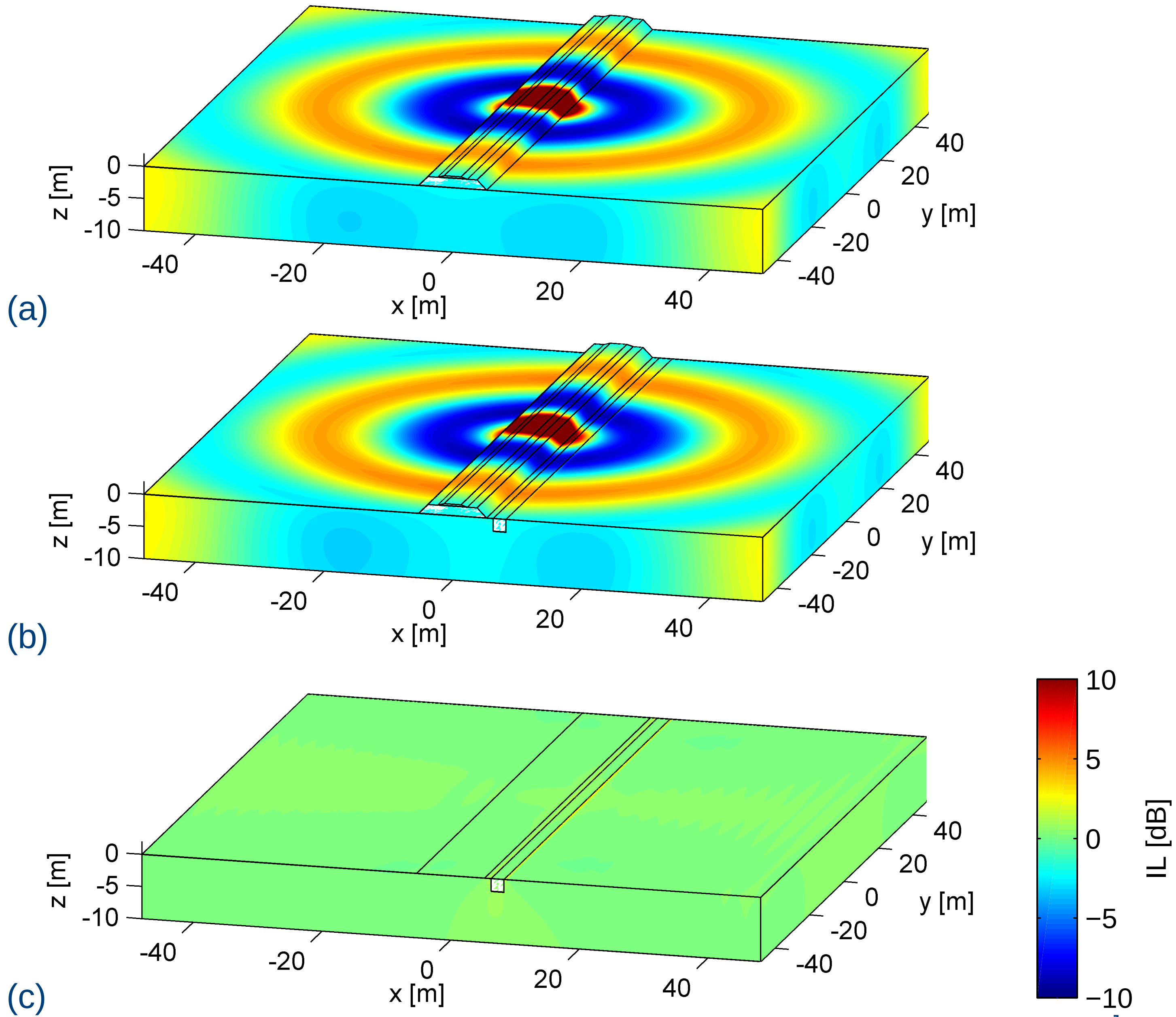
C_s $[\mathrm{m/s}]$	C_p $[\mathrm{m/s}]$	ν $[-]$	$\beta_\mathrm{s} = \beta_\mathrm{p}$ $[-]$	ρ $[\mathrm{kg/m^3}]$
550	950	0.25	0.05	2000

- Insertion loss IL_z:

$$\mathrm{IL}_z = 10 \log_{10} \frac{|u_z^{\mathrm{ref}}|^2}{|u_z|^2} \qquad [\mathrm{dB}]$$

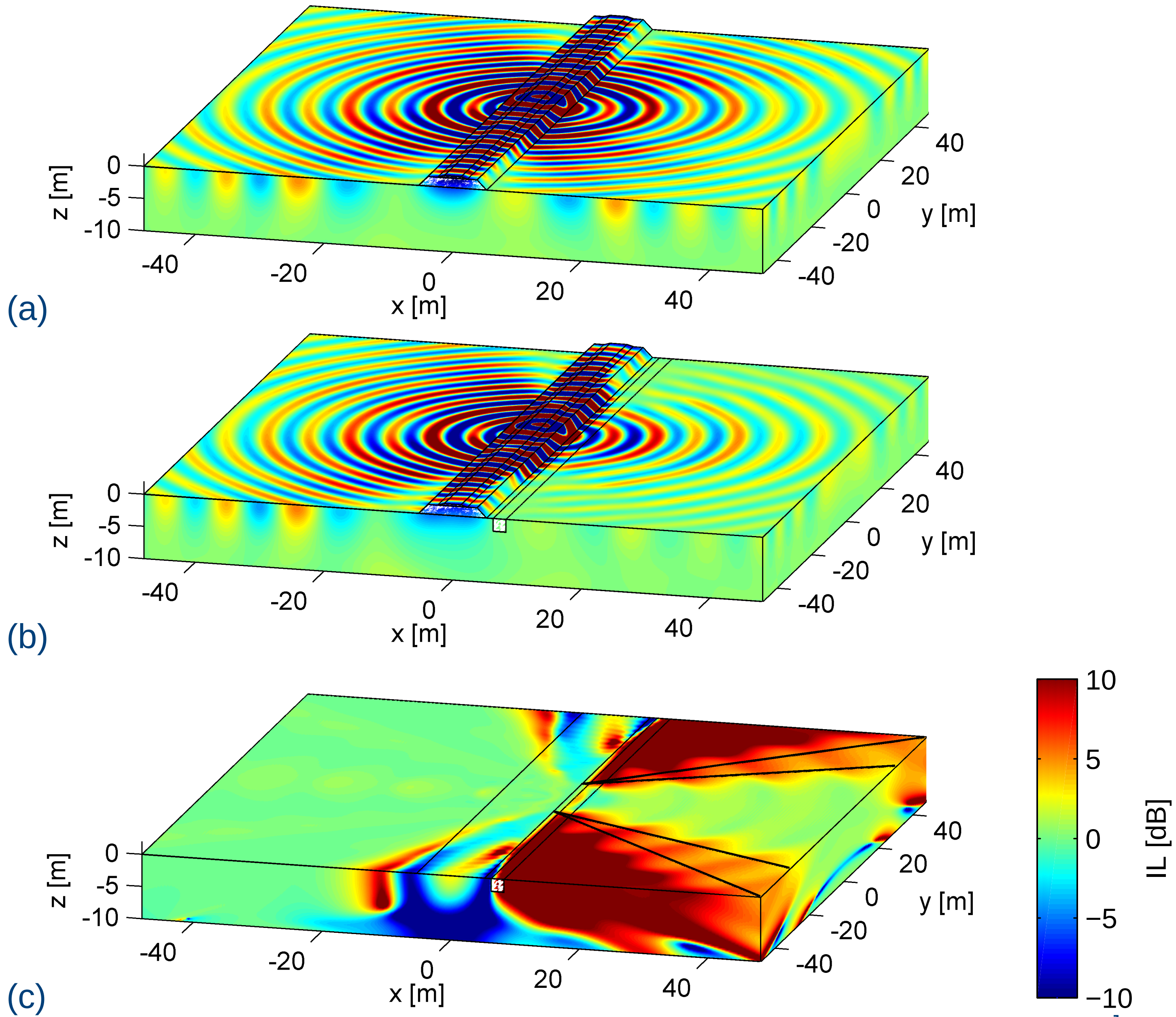
Insertion loss (x, y, z)

- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of stiffening next to the track at $5\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).



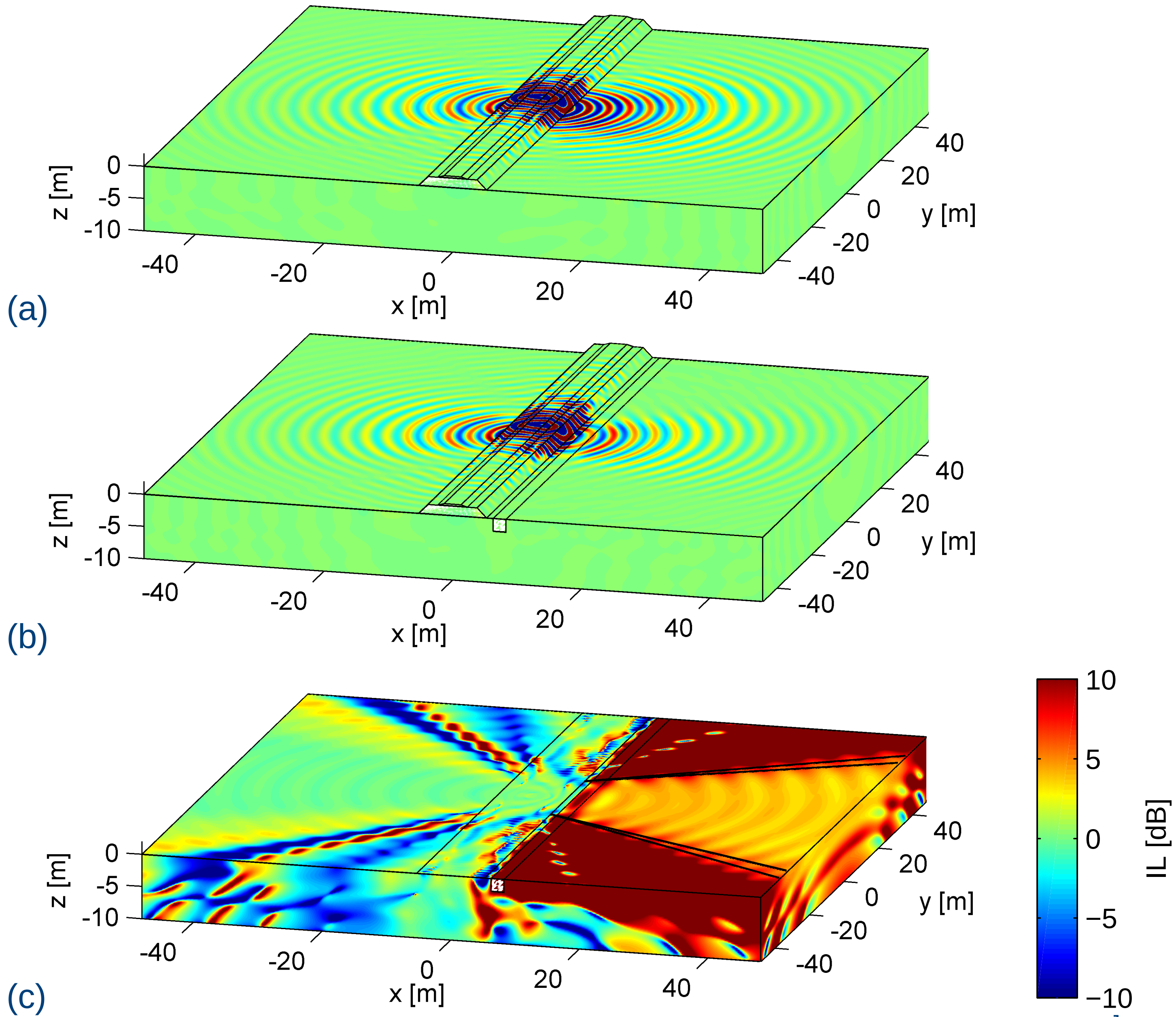
Insertion loss (x, y, z)

- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of stiffening next to the track at $30\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).



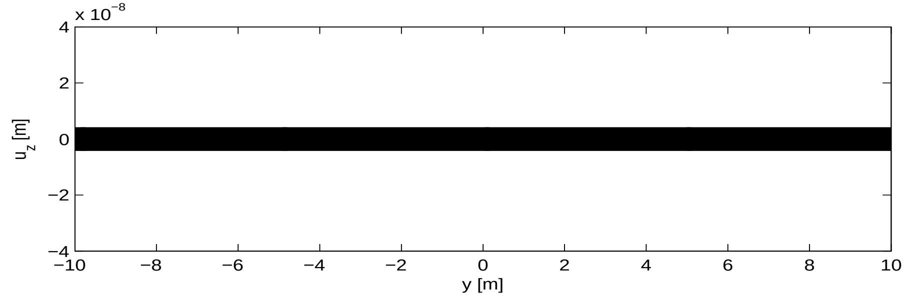
Insertion loss (x, y, z)

- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of stiffening next to the track at $60\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).

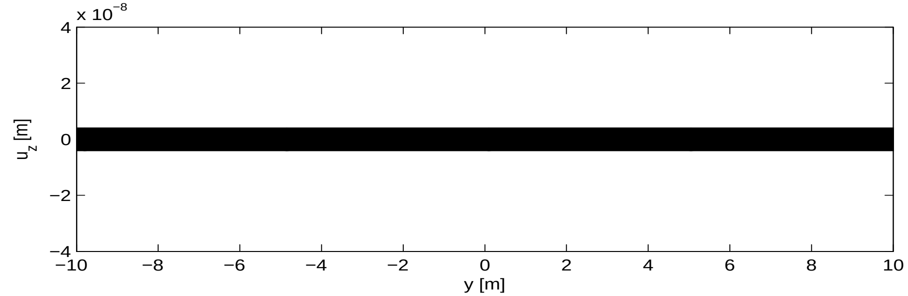


Beam behaviour at $30\,\mathrm{Hz}$

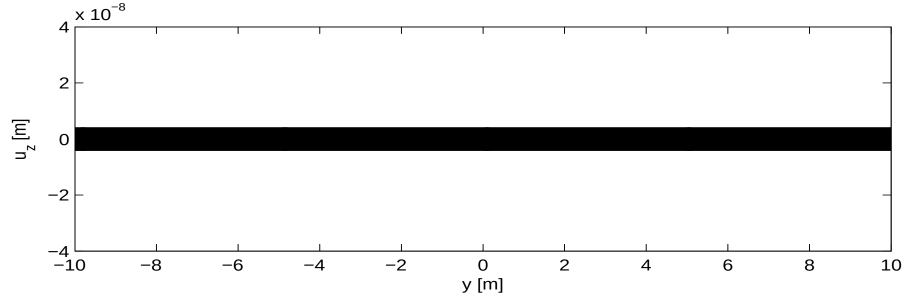
- $\lambda_y = \infty$



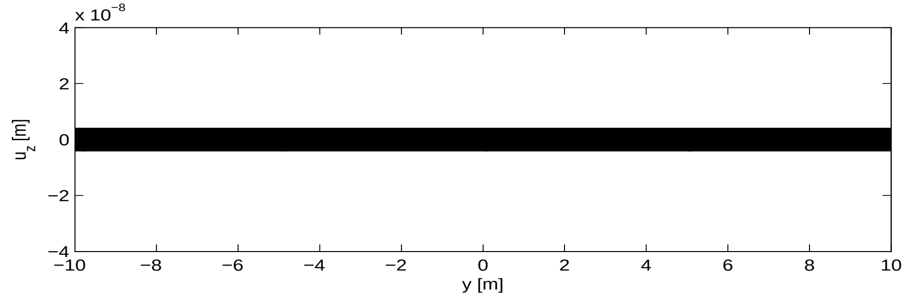
- $\lambda_y = \lambda_\mathrm{b}$



- $\lambda_y = 2/3\lambda_\mathrm{b}$



- $\lambda_y = 1/3\lambda_\mathrm{b}$



Beam behaviour

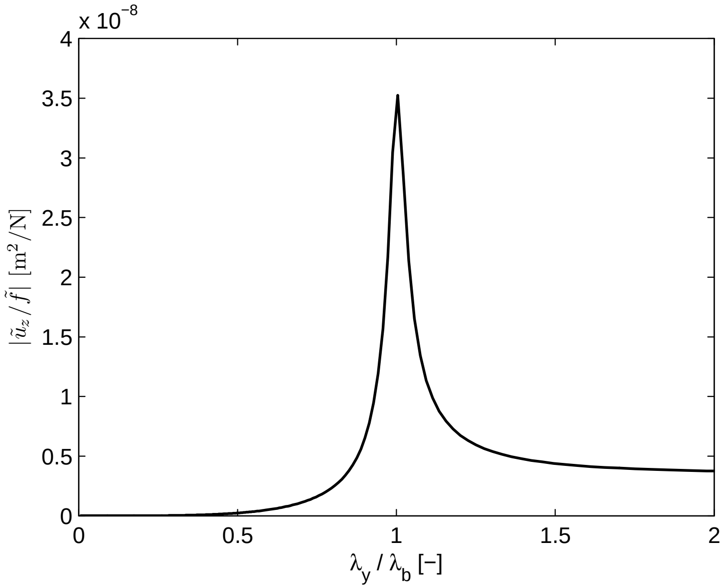
- Euler-Bernoulli beam theory in (λ_y, ω)-domain:

$$\left(-\rho A\omega^2 + EI\left(\frac{2\pi}{\lambda_y}\right)^4\right)\tilde{u}_z(\lambda_y,\omega) = \tilde{f}(\lambda_y,\omega)$$

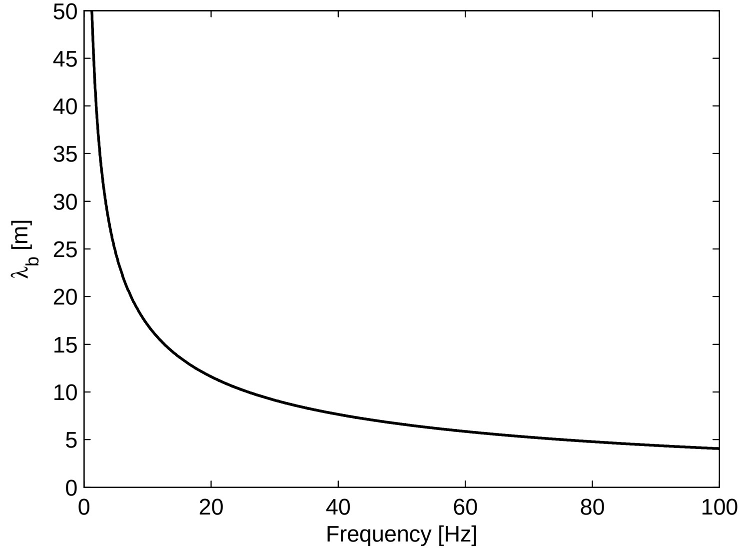
- Wavelength λ_b corresponding to a free bending wave:

$$\lambda_\mathrm{b} = \frac{2\pi}{\sqrt{\omega}}\left(\frac{EI}{\rho A}\right)^{1/4}$$

- (a) $|\tilde{u}_z/\tilde{f}|$ at $30\,\mathrm{Hz}$ vs. $\lambda_y/\lambda_\mathrm{b}$ and (b) λ_b vs. frequency
  - $\lambda_y > \lambda_\mathrm{b}$: amplitude of the propagating wave is determined by the inertia
  - $\lambda_y < \lambda_\mathrm{b}$: amplitude of the propagating wave is determined by the bending stiffness



(a)



(b)

Interaction of Rayleigh wave and bending waves

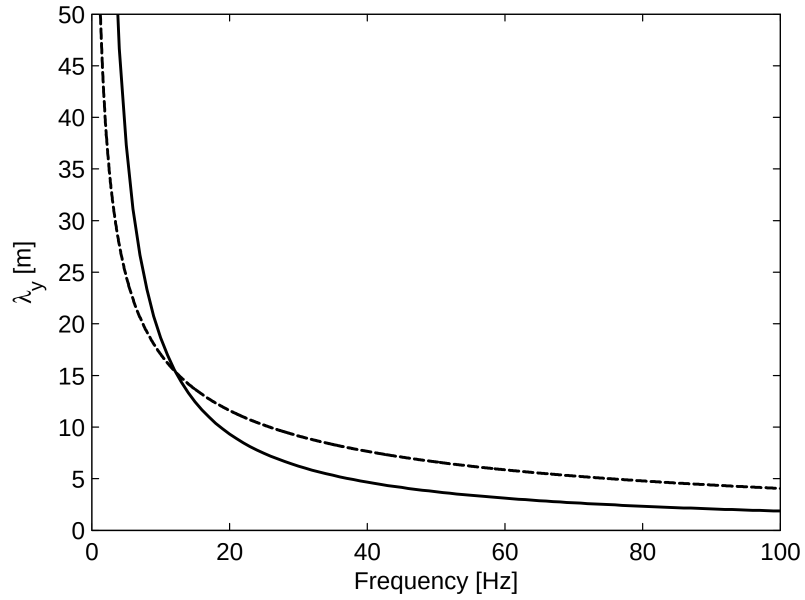
- Rayleigh wave dispersion curve (solid line):

$$\lambda_{\mathrm{R}} = 2\pi \frac{C_{\mathrm{R}}}{\omega}$$

$$\lambda_y = \frac{\lambda_{\mathrm{R}}}{\sin\theta} \Rightarrow \lambda_{\mathrm{R}} \leq \lambda_y \leq \infty$$

- Free bending wave dispersion curve (dashed line):

$$\lambda_{\mathrm{b}} = \frac{2\pi}{\sqrt{\omega}} \left(\frac{EI}{\rho A}\right)^{1/4}$$



- $\lambda_{\mathrm{b}} < \lambda_{\mathrm{R}}$: wavefield is transmitted
- $\lambda_{\mathrm{b}} > \lambda_{\mathrm{R}}$: wavefield is partially transmitted, partially blocked
  - $\lambda_y > \lambda_{\mathrm{b}}$: transmitted
  - $\lambda_y < \lambda_{\mathrm{b}}$: blocked

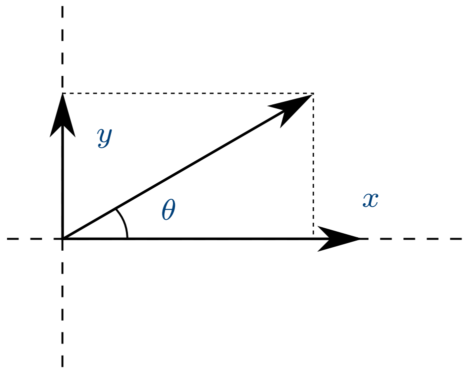
- Critical frequency f_{c} ($\lambda_{\mathrm{R}} = \lambda_{\mathrm{b}}$):

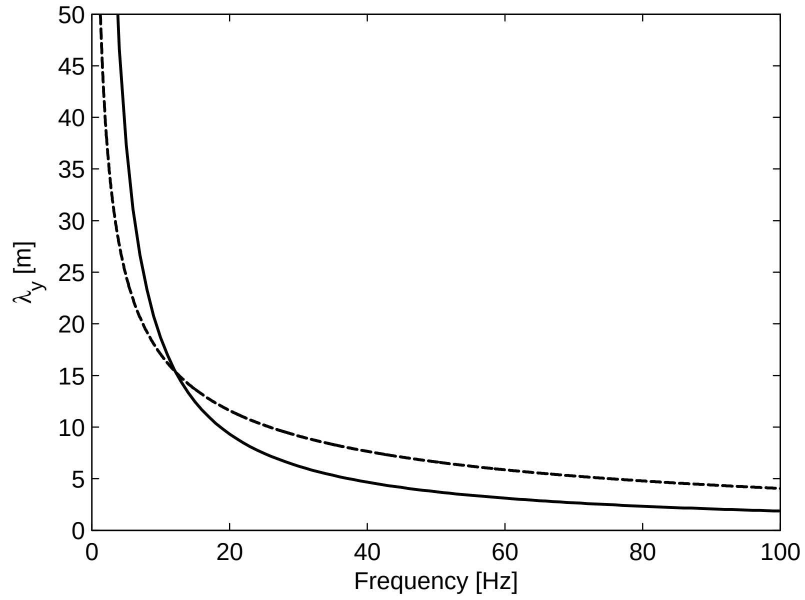
$$\boxed{f_{\mathrm{c}} = \frac{C_{\mathrm{R}}^2}{2\pi}\sqrt{\frac{\rho A}{EI}} = \frac{C_{\mathrm{R}}^2}{2\pi h}\sqrt{\frac{12\rho}{E}}}$$

Interaction of Rayleigh wave and bending waves

- Wave propagation direction θ:

$$\lambda_y = \frac{\lambda_\mathrm{R}}{\sin\theta}$$
$$\lambda_x = \frac{\lambda_\mathrm{R}}{\cos\theta}$$





- Critical angle θ_c ($\lambda_y = \lambda_\mathrm{b}$):

$$\boxed{\sin\theta_\mathrm{c} = \frac{C_\mathrm{R}}{\sqrt{\omega}}\left(\frac{\rho A}{EI}\right)^{1/4} = \frac{C_\mathrm{R}}{\sqrt{\omega h}}\left(\frac{12\rho}{E}\right)^{1/4}}$$

- $f = 30\,\mathrm{Hz}$

$$\sin\theta_\mathrm{c} = \frac{\lambda_\mathrm{R}}{\lambda_\mathrm{b}} = \frac{6.22}{9.14}$$
$$\Rightarrow \theta_\mathrm{c} = 42.9^\circ$$

- $f = 60\,\mathrm{Hz}$

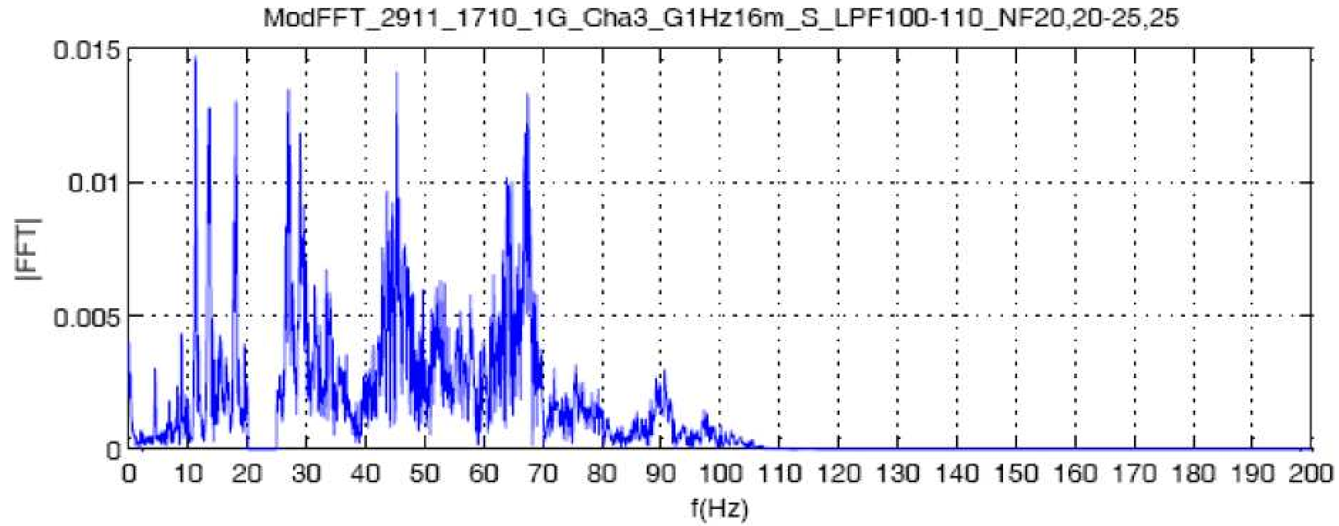
$$\sin\theta_\mathrm{c} = \frac{\lambda_\mathrm{R}}{\lambda_\mathrm{b}} = \frac{3.11}{5.88}$$
$$\Rightarrow \theta_\mathrm{c} = 32.1^\circ$$

Location

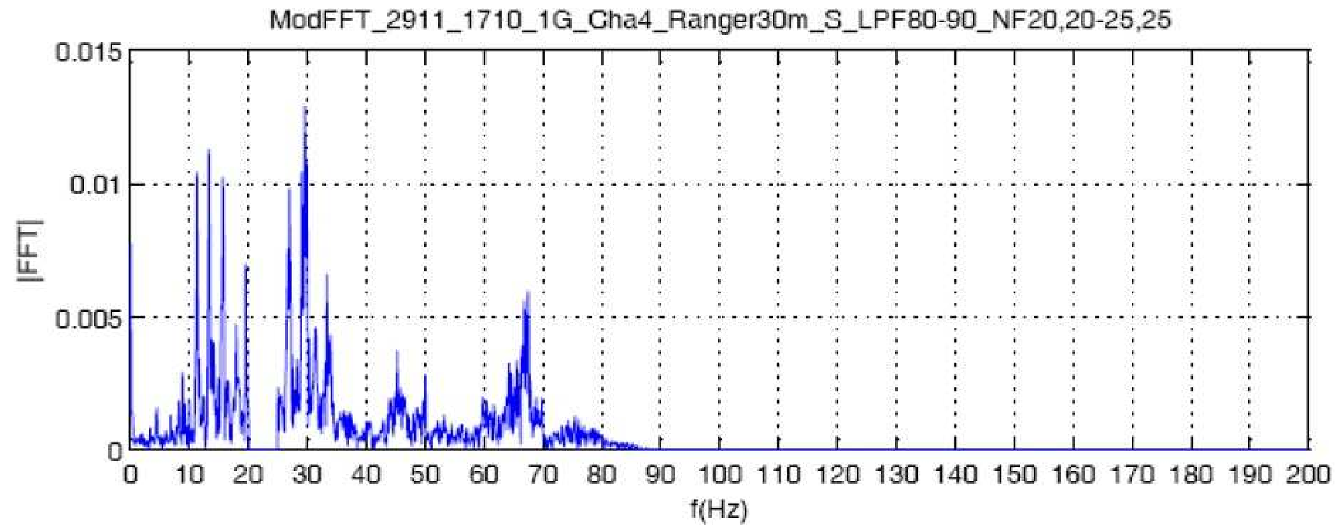
- Site located between Barcelona and Girona, next to a high speed line and a conventional line

Measurement campaign (ADIF & CEDEX)

- Free field velocity at 16 m from the track due to the passage of a medium distance train

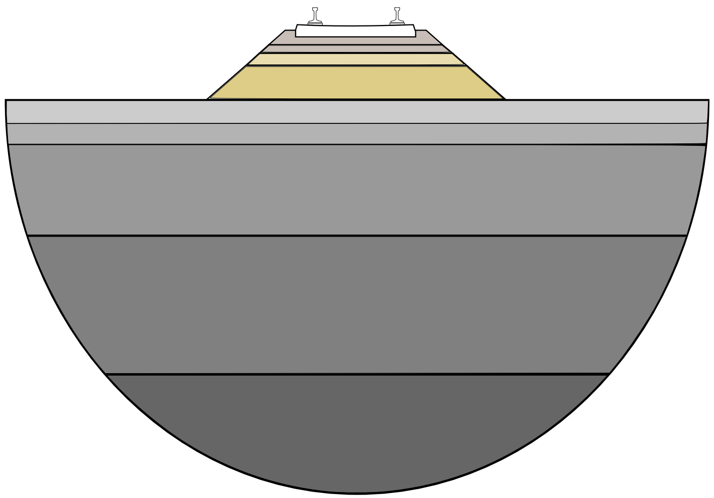


- Free field velocity at 30 m from the track due to the passage of a medium distance train



Horizontally layered visco-elastic halfspace

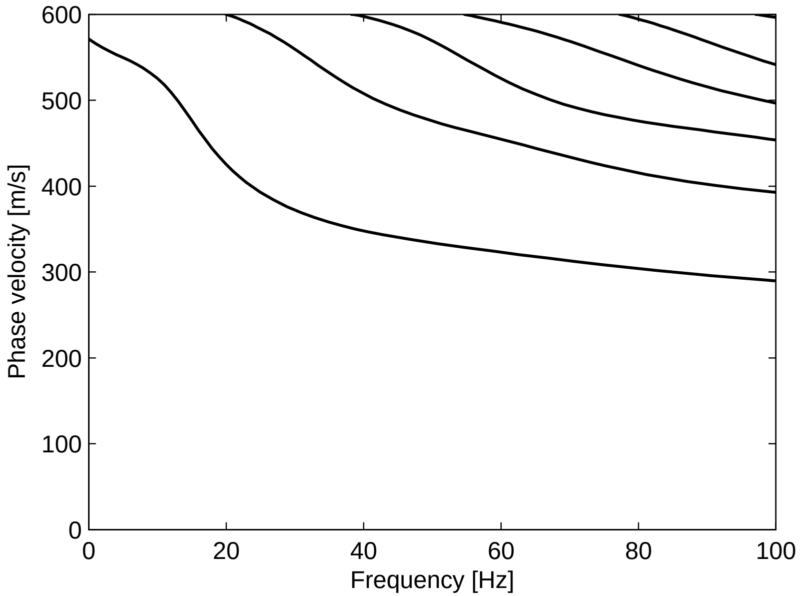
- Coarse gravel immersed in sandy clay (alluvial deposits) on sandstone, conglomerates and clay (tertiary deposits).
- CEDEX has performed geotechnical tests at the proposed test site (active and passive SASW-tests, seismic refraction tests) in order to obtain the dynamic soil characteristics.
- A simplified model of the soil is finally determined.



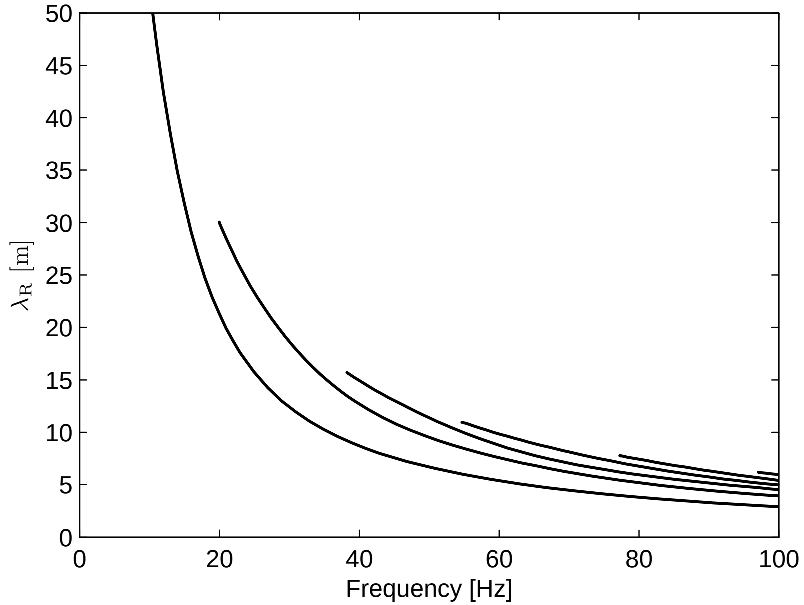
Layer	Thickness [m]	C_s [m/s]	C_p [m/s]	β_s [−]	β_p [−]	ρ [kg/m^3]
1	1	275	740	0.025	0.025	2000
2	1	325	740	0.025	0.025	2000
3	4	380	1450	0.025	0.025	2000
4	7	470	2280	0.025	0.025	2000
5	∞	600	2580	0.025	0.025	2000

Rayleigh wave dispersion curves

- Dispersive behaviour due to the variation of soil properties with depth
- Multiple modes appear with cut-on frequencies
- (a) Phase velocity C_{R} and (b) wavelength $\lambda_R = C_{\mathrm{R}}/f$ vs. frequency f



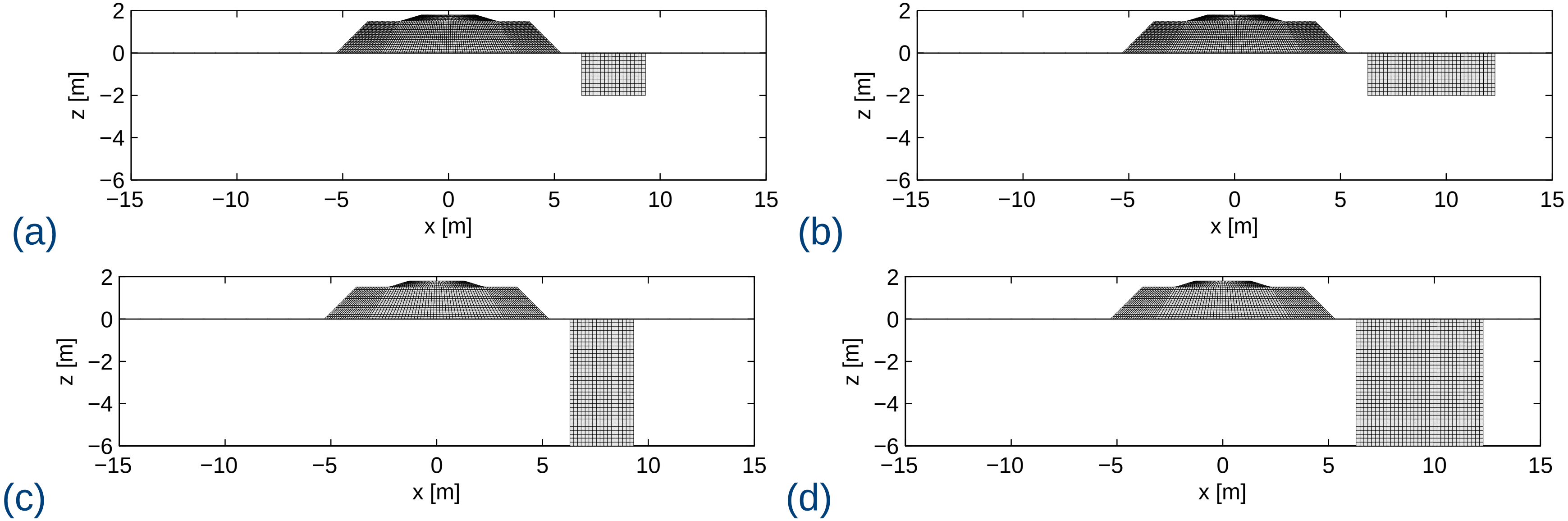
(a)



(b)

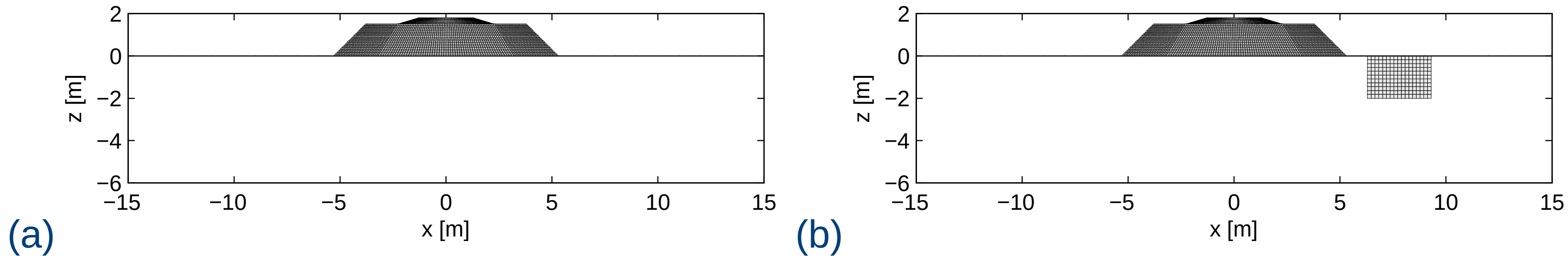
Four variants

Variant	Part	Width	Thickness
A	Soil layer 1	3 m	1 m
	Soil layer 2	3 m	1 m
B	Soil layer 1	6 m	1 m
	Soil layer 2	6 m	1 m
C	Soil layer 1	3 m	1 m
	Soil layer 2	3 m	1 m
	Soil layer 3	3 m	4 m
D	Soil layer 1	6 m	1 m
	Soil layer 2	6 m	1 m
	Soil layer 3	6 m	4 m

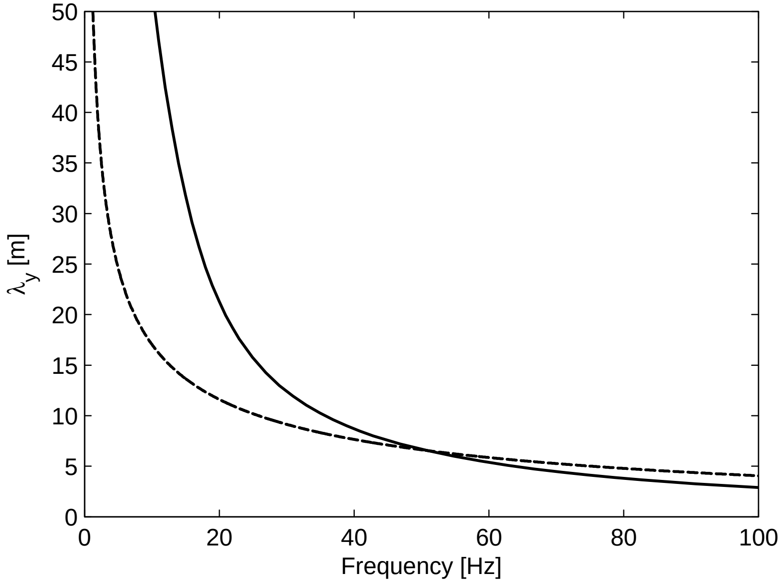


Variant A

- Rectangular cross section: $w = 3\,\mathrm{m}$, $h = 2\,\mathrm{m}$

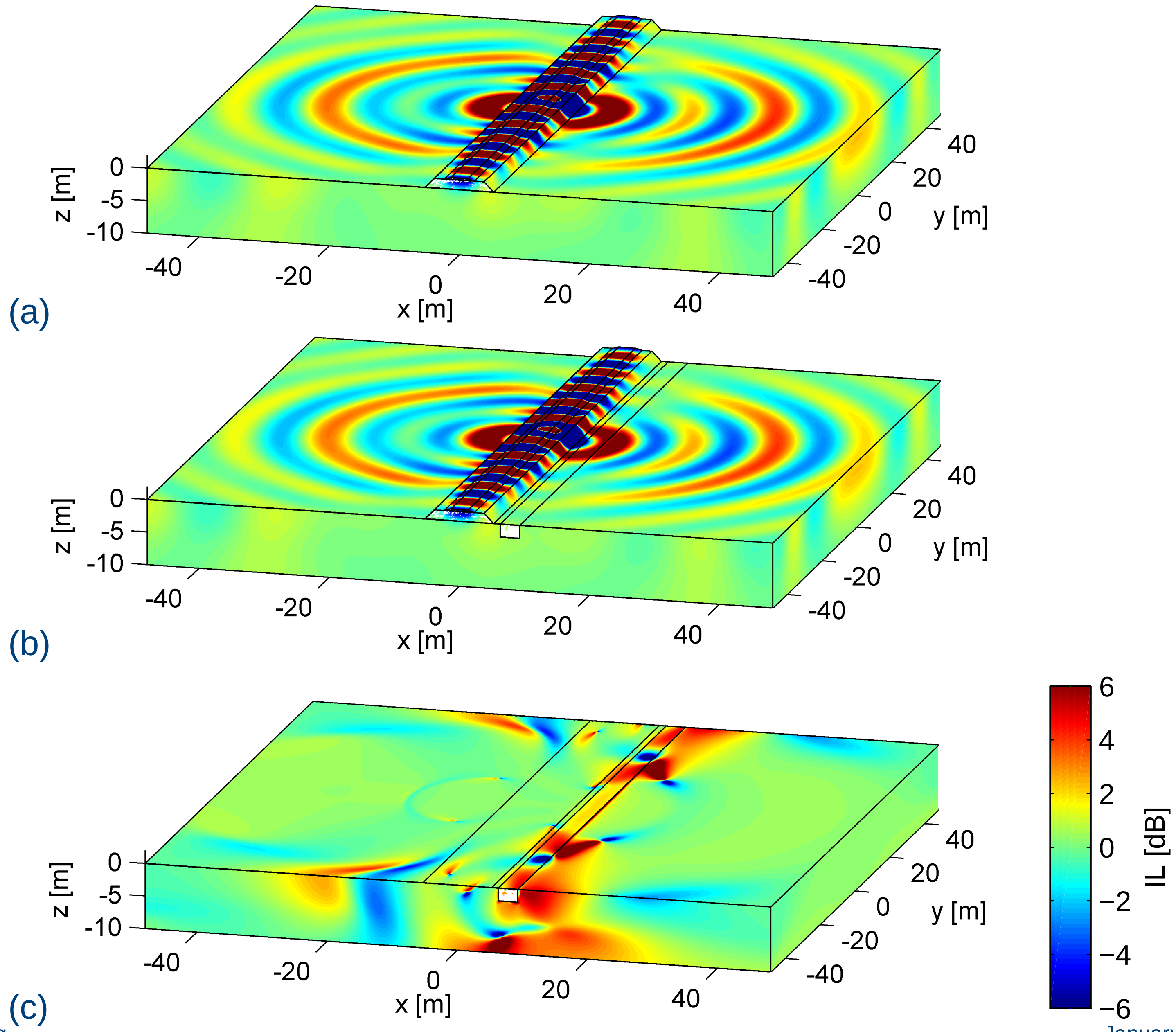


- Interaction of Rayleigh wave and bending wave:
  - Rayleigh wave dispersion curve (solid line)
  - Free bending wave dispersion curve (dashed line)

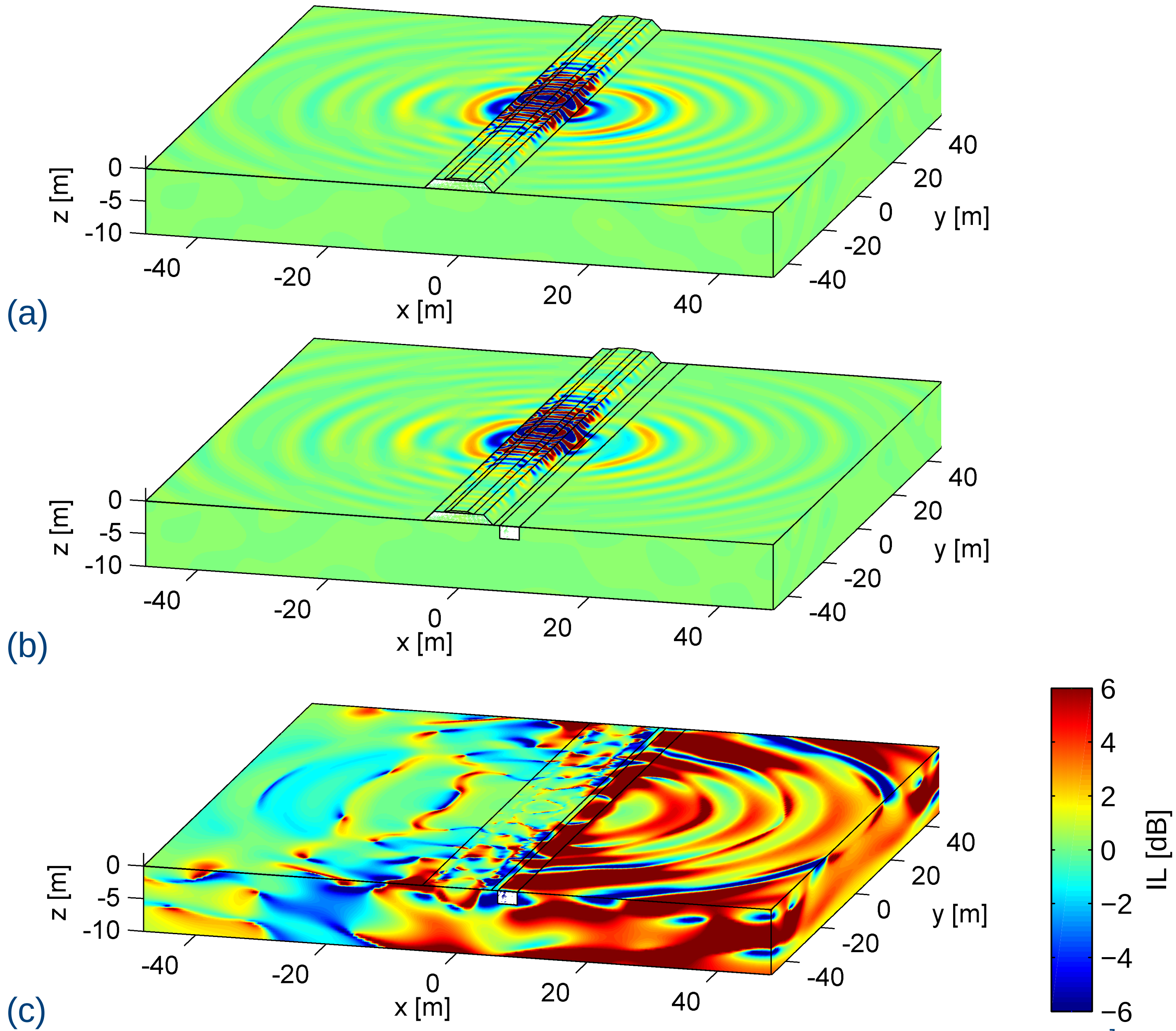


Insertion loss (x, y, z)

- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of stiffening next to the track at $30\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).

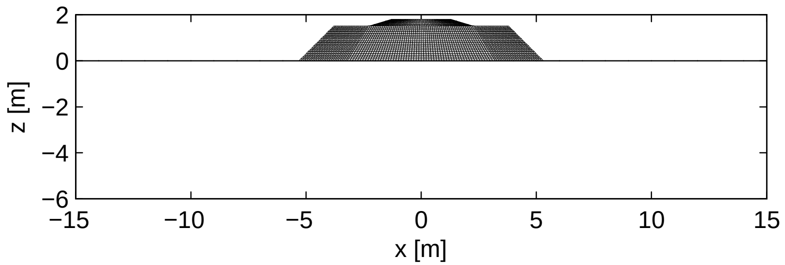


- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of stiffening next to the track at $60\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).

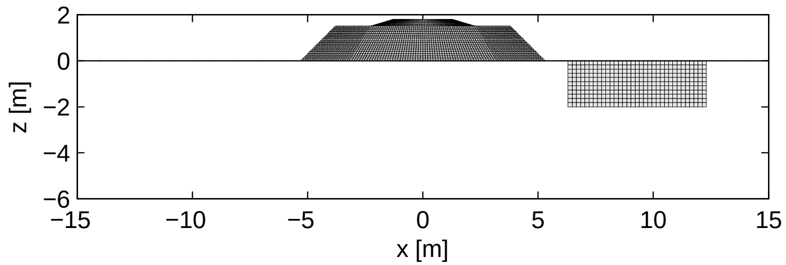


Variant B

- Rectangular cross section: $w = 6\,\mathrm{m}$, $h = 2\,\mathrm{m}$

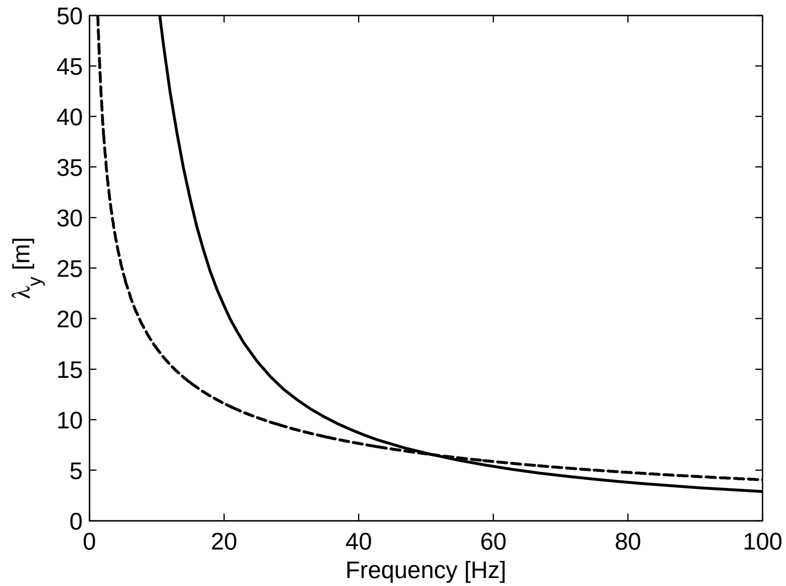


(a)



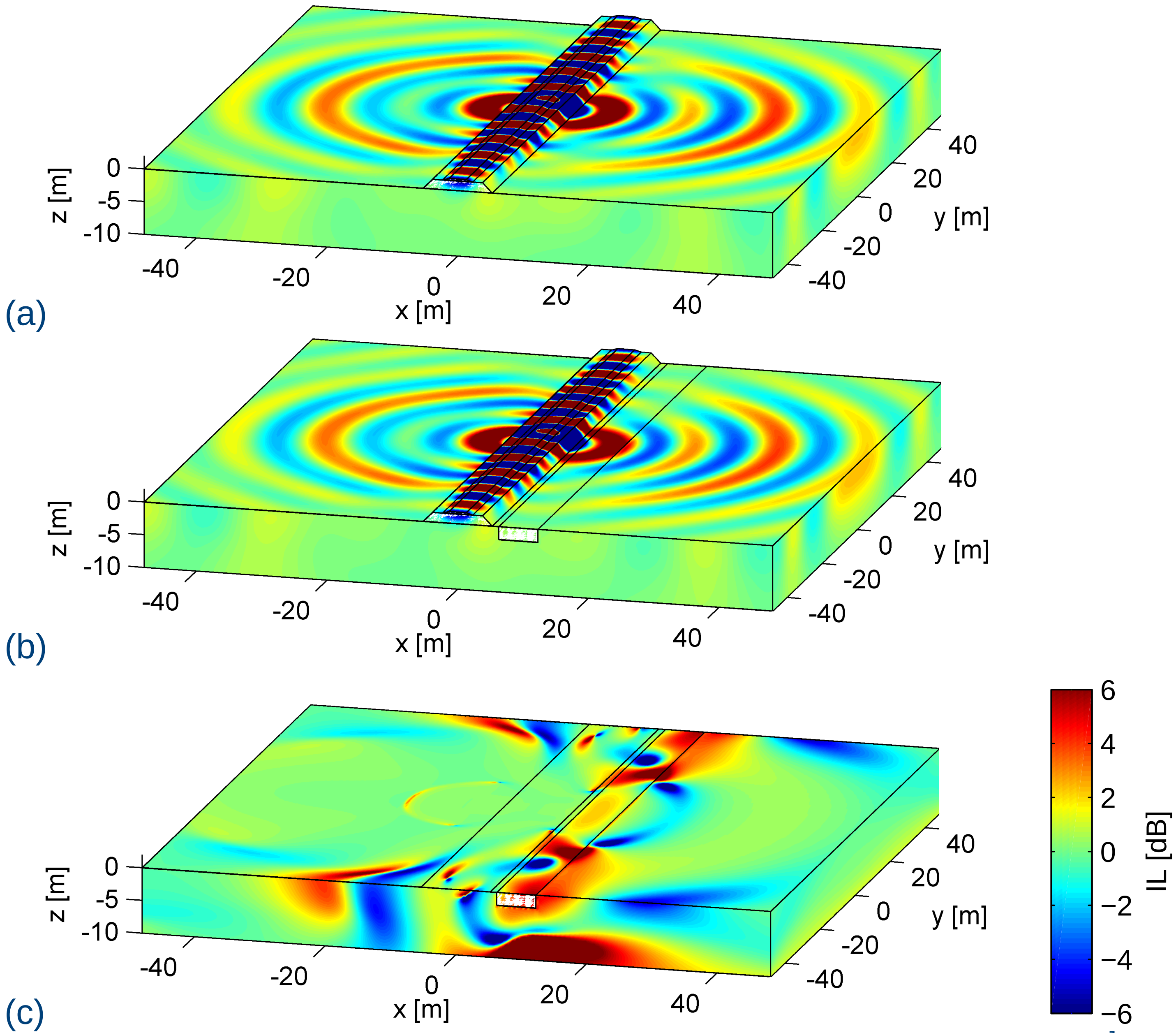
(b)

- Interaction of Rayleigh wave and bending wave:
  - Rayleigh wave dispersion curve (solid line)
  - Free bending wave dispersion curve (dashed line)



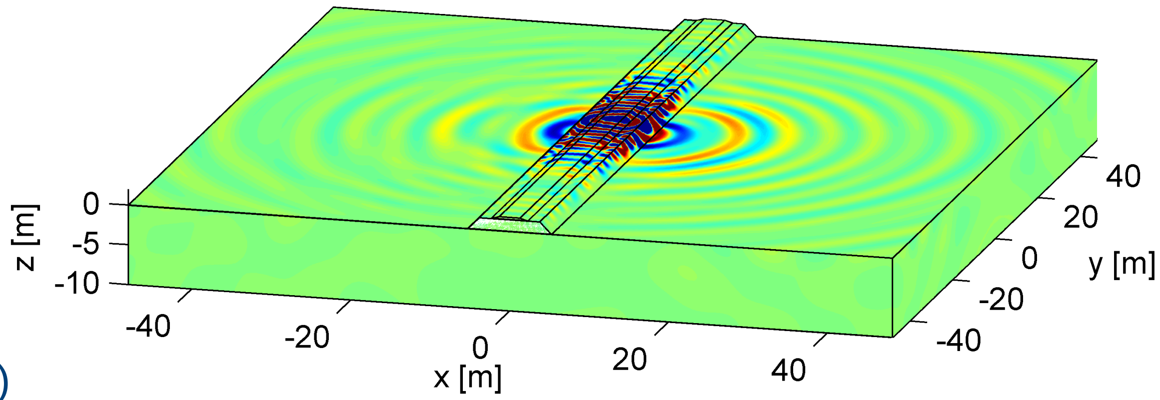
Insertion loss (x, y, z)

- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of stiffening next to the track at $30\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).

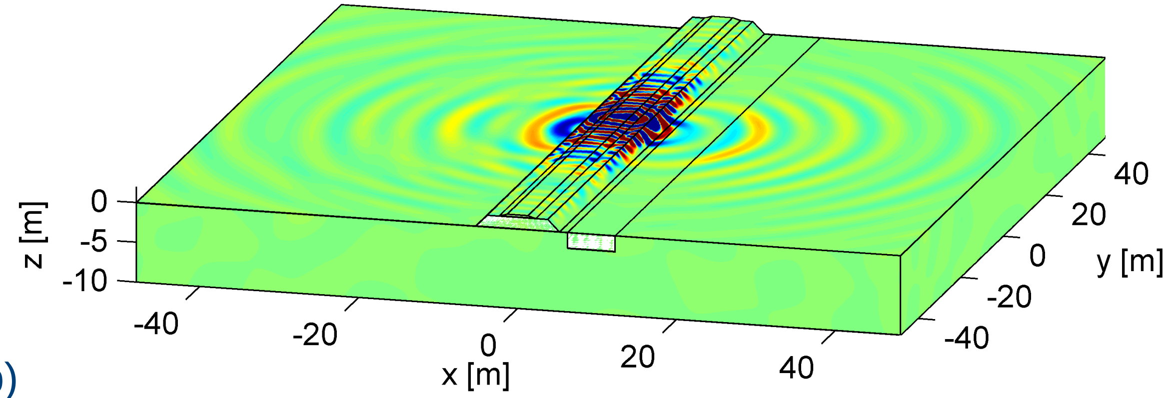


Insertion loss (x, y, z)

- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of stiffening next to the track at $60\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).



(a)



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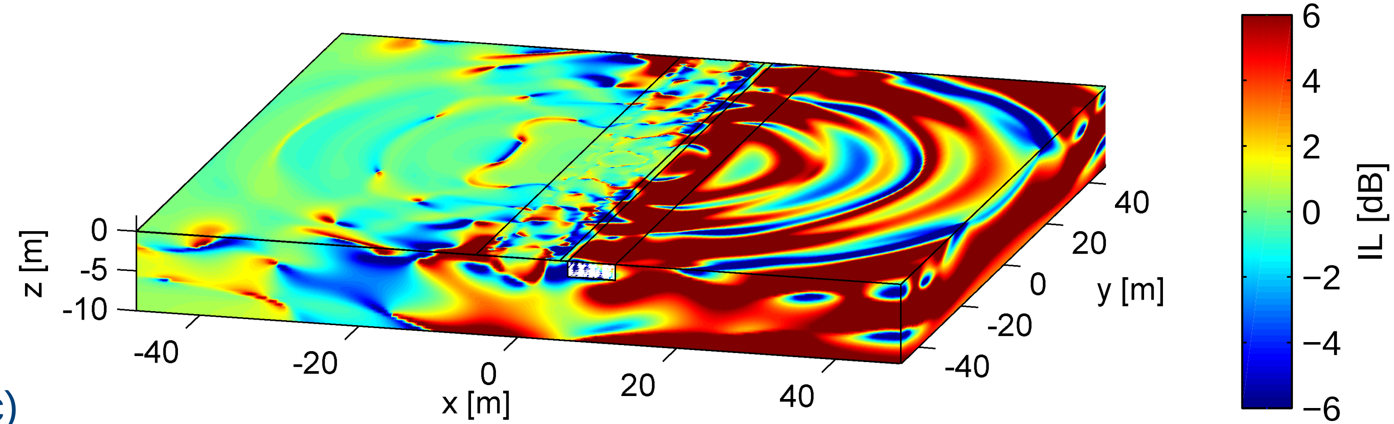
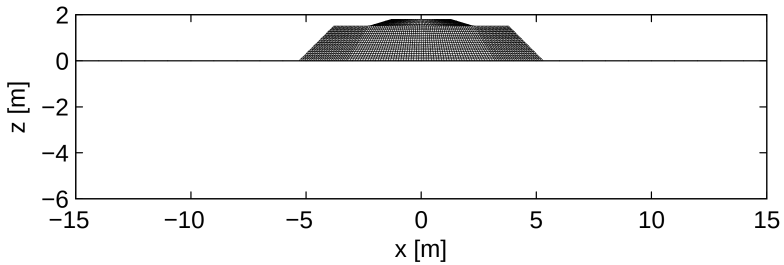


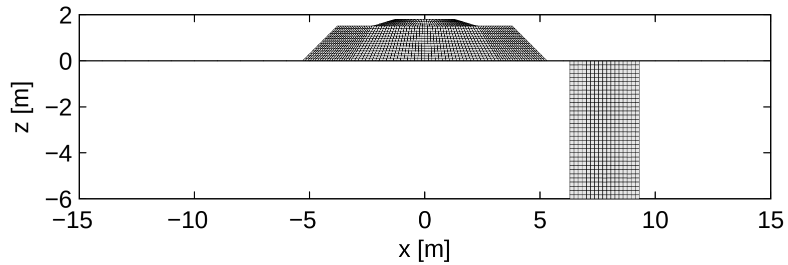
(c)

Variant C

- Rectangular cross section: $w = 3\,\mathrm{m}$, $h = 6\,\mathrm{m}$

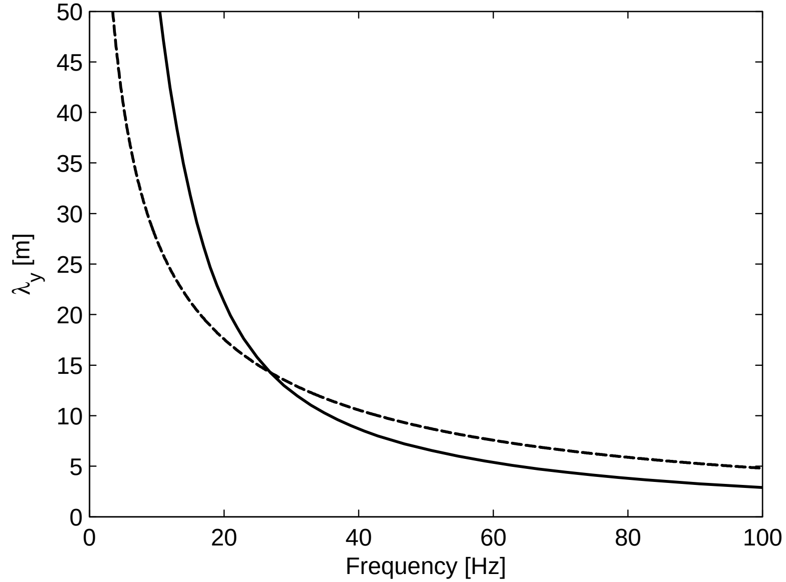


(a)



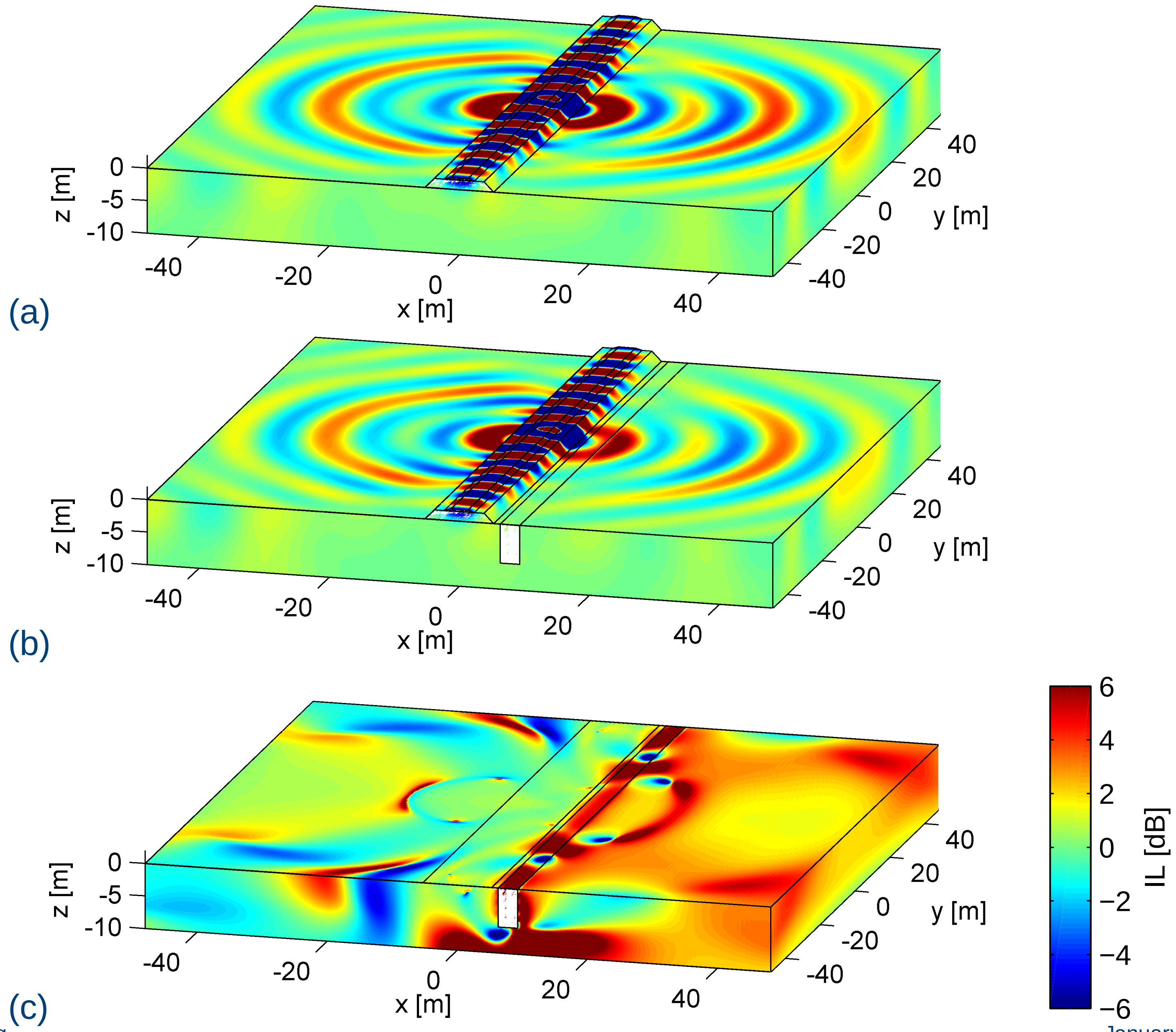
(b)

- Interaction of Rayleigh wave and bending wave:
  - Rayleigh wave dispersion curve (solid line)
  - Free bending wave dispersion curve (dashed line)



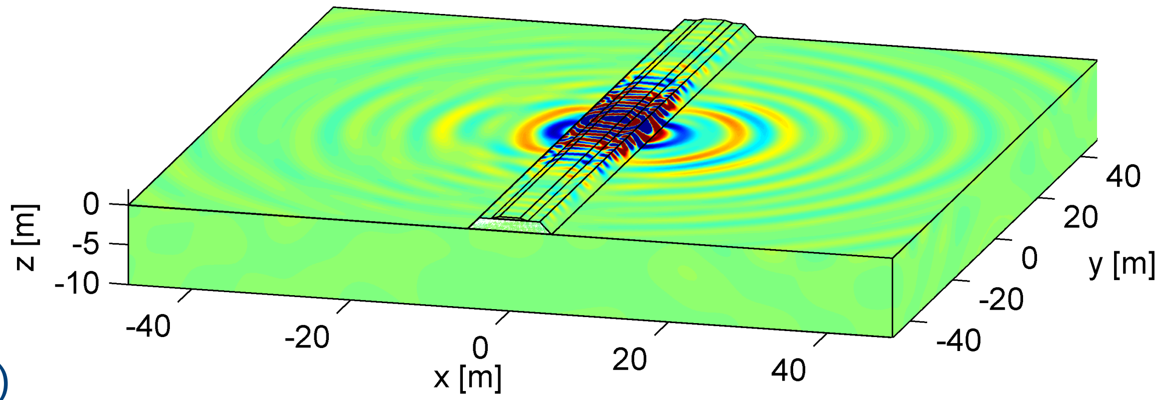
Insertion loss (x, y, z)

- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of stiffening next to the track at $30\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).

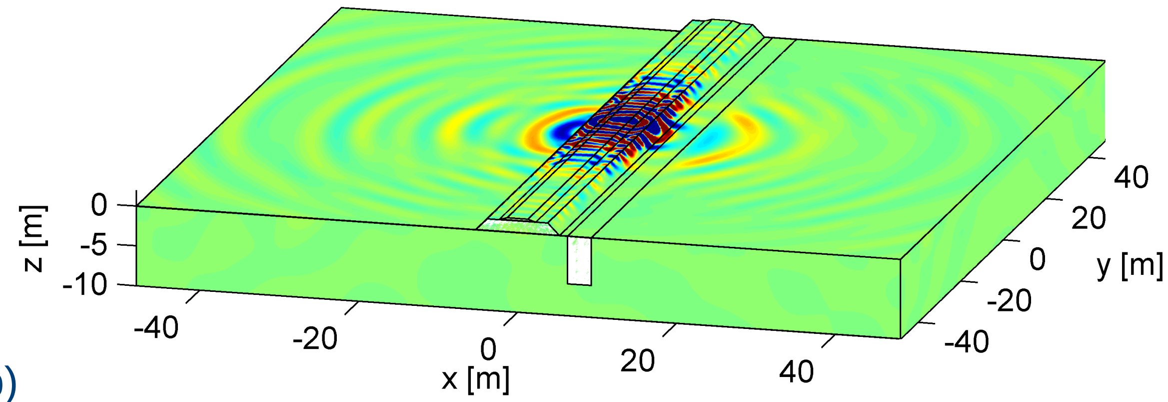


Insertion loss (x, y, z)

- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of stiffening next to the track at $60\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).



(a)



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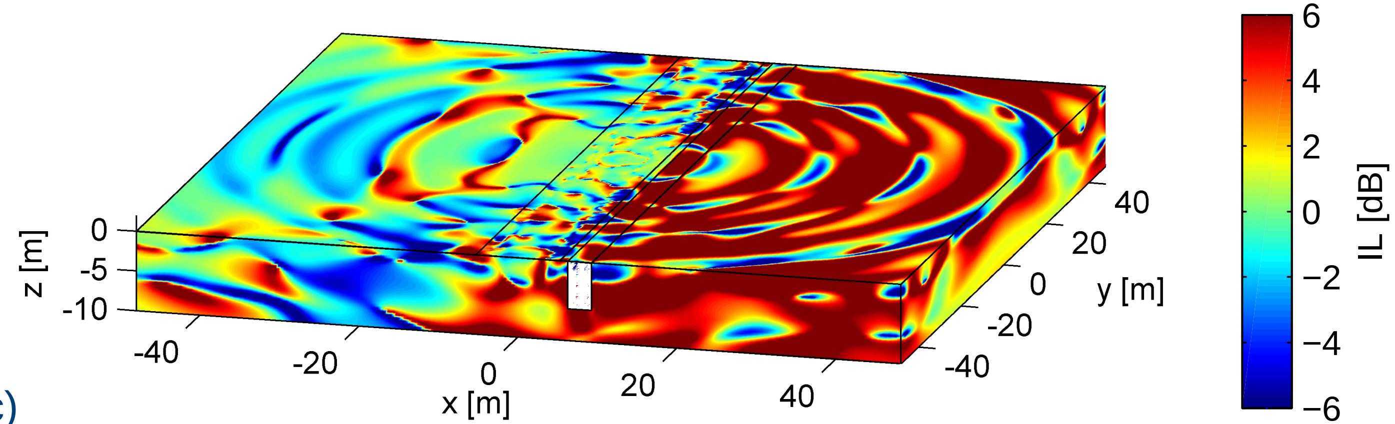
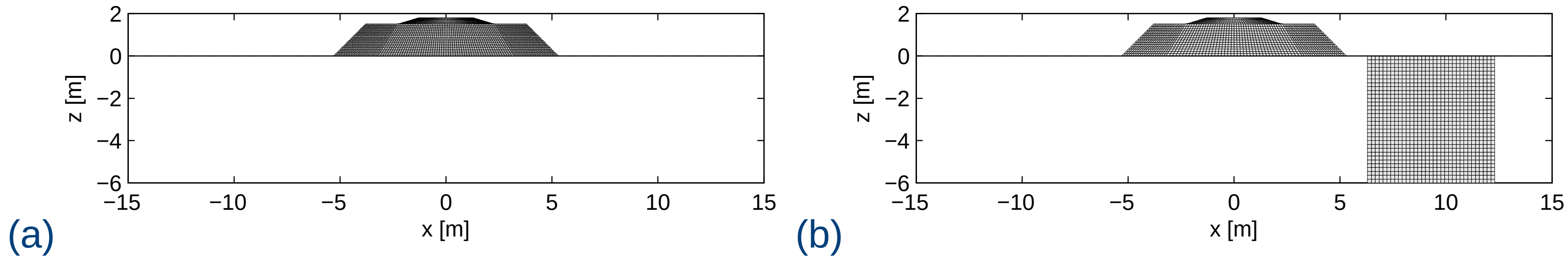


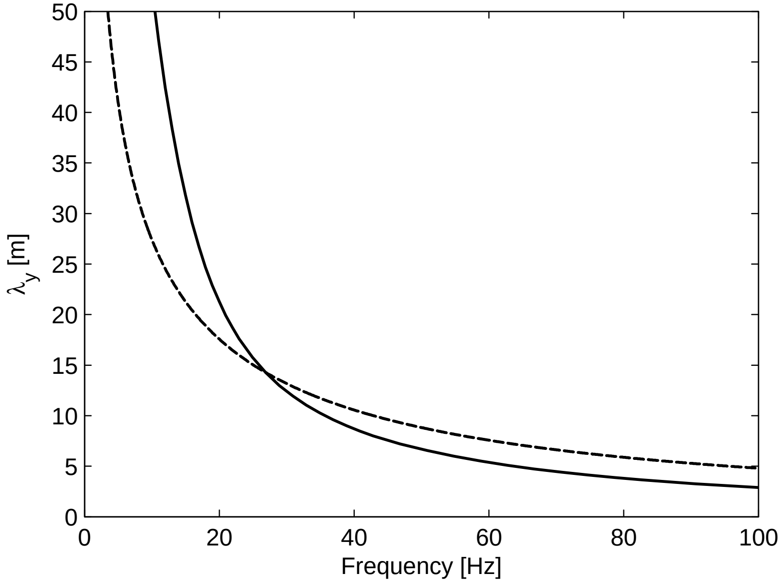
(c)

Variant D

- Rectangular cross section: $w = 6\,\mathrm{m}$, $h = 6\,\mathrm{m}$

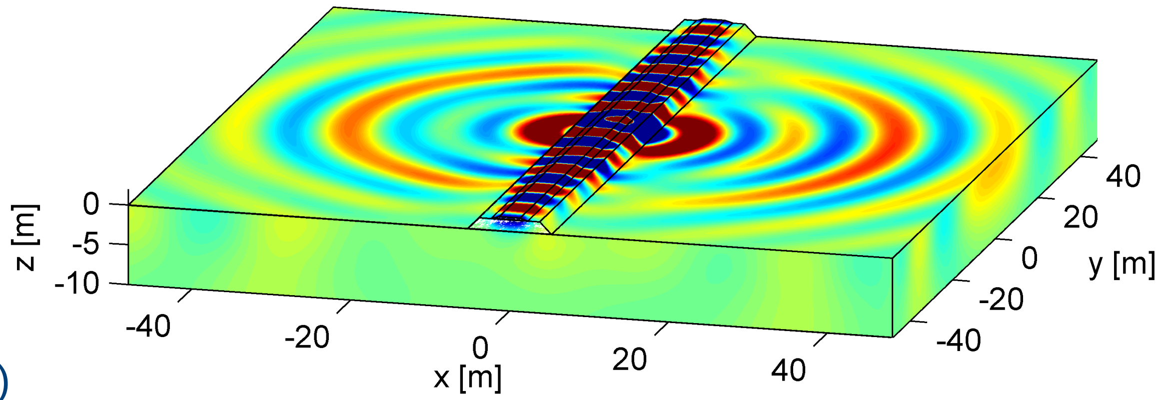


- Interaction of Rayleigh wave and bending wave:
  - Rayleigh wave dispersion curve (solid line)
  - Free bending wave dispersion curve (dashed line)

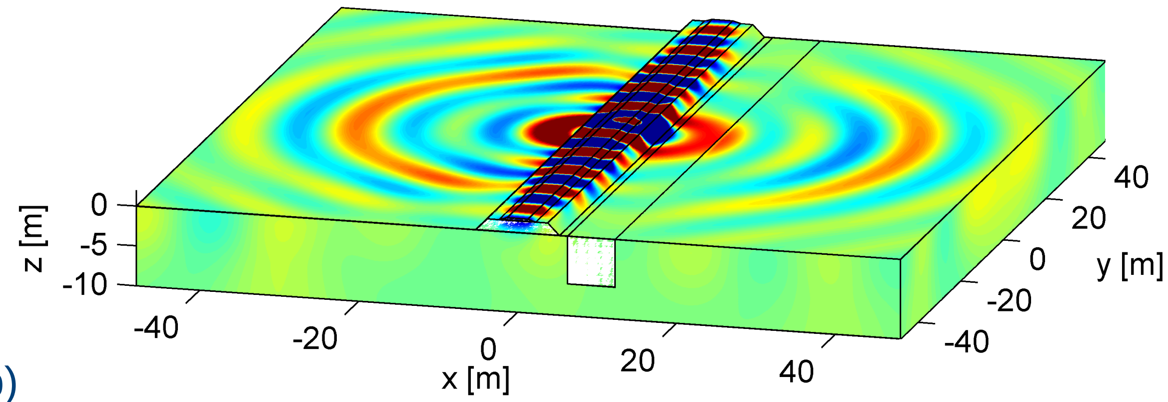


Insertion loss (x, y, z)

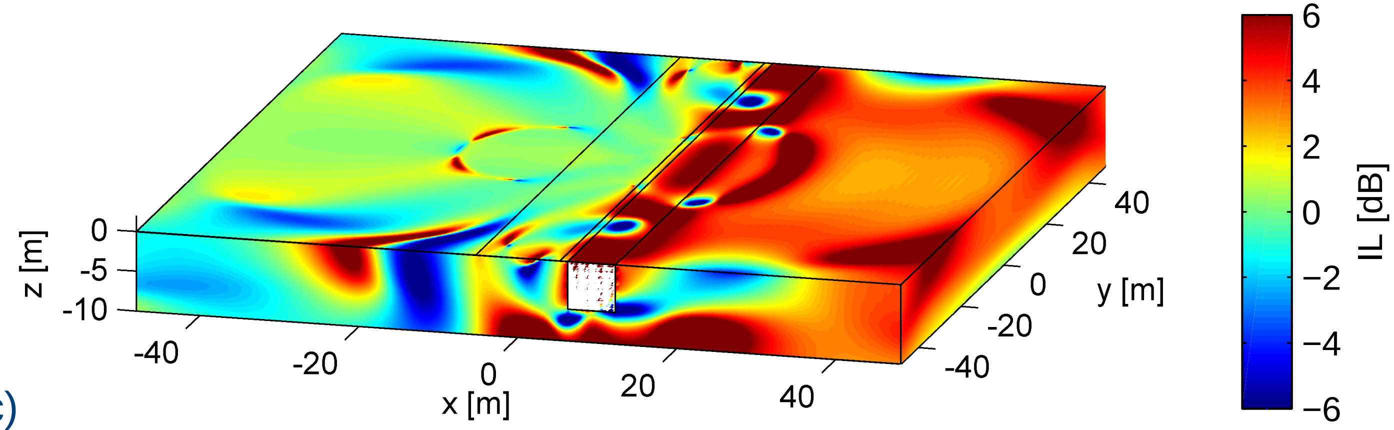
- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of stiffening next to the track at $30\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).



(a)



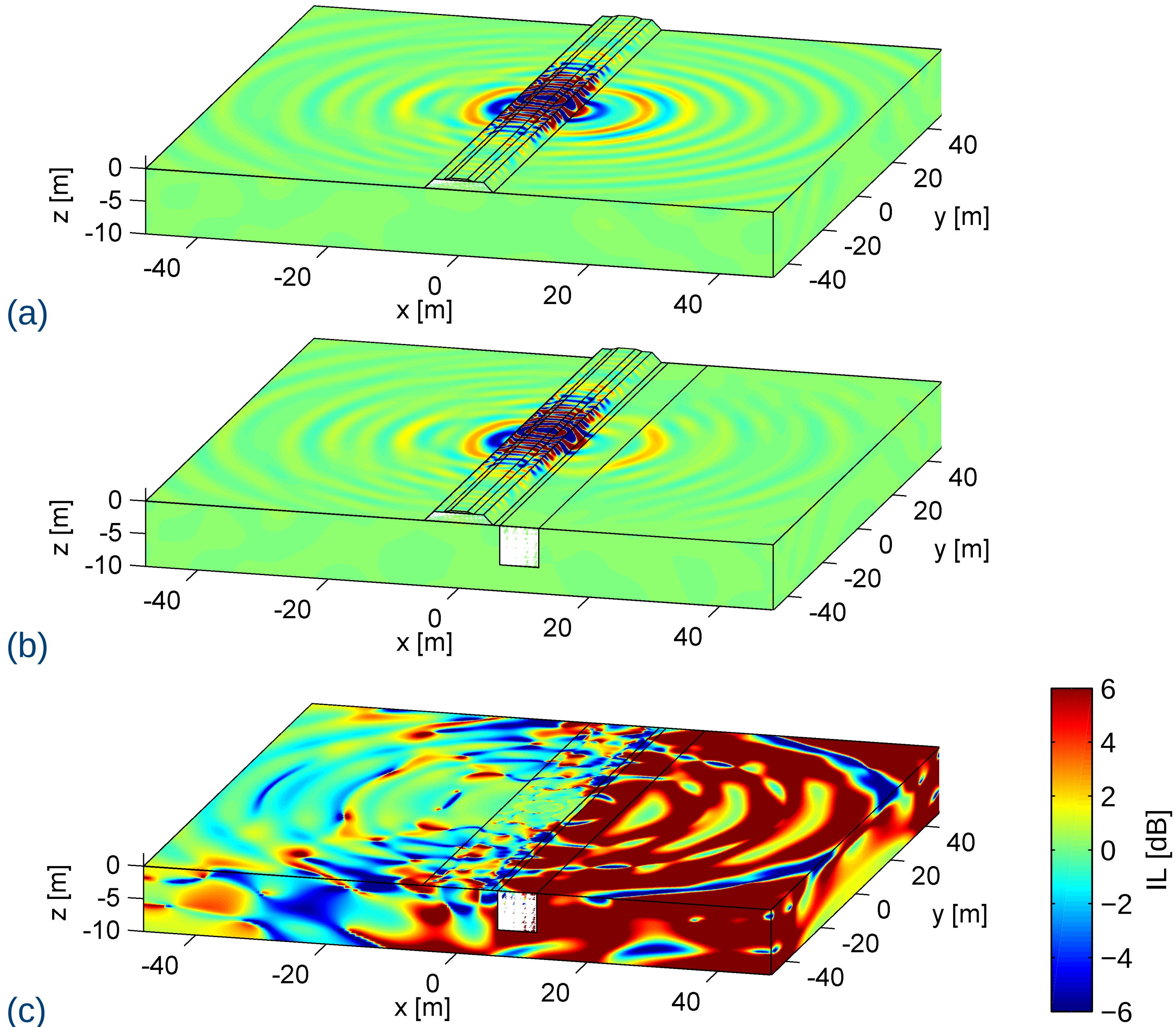
(b)



(c)

Insertion loss (x, y, z)

- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of stiffening next to the track at $60\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).



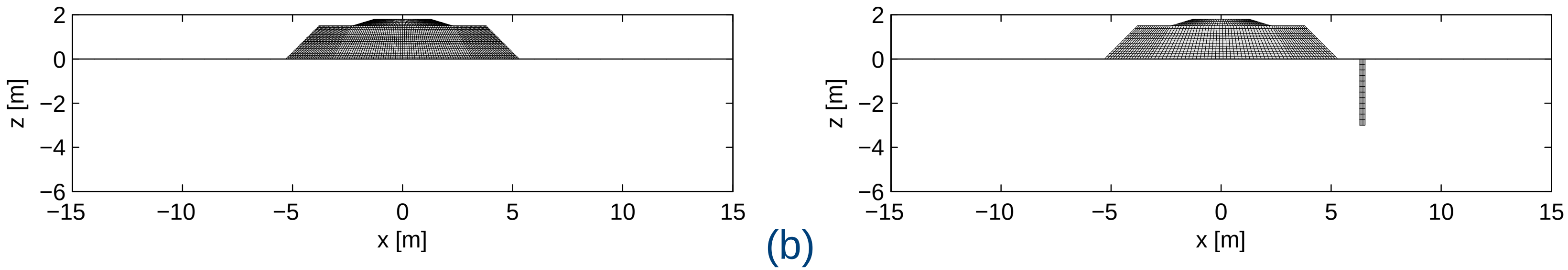
(a) (b) (c)

Geofoam filled trench

- Rule of thumb states that (open) trenches only effective when their depth is 0.6 times the wavelength in the soil:

$$f_{\mathrm{c}} = 0.6\frac{C}{h}$$

- Rectangular cross section: $w = 0.25\,\mathrm{m}$, $h = 3\,\mathrm{m}$



(a) (b)

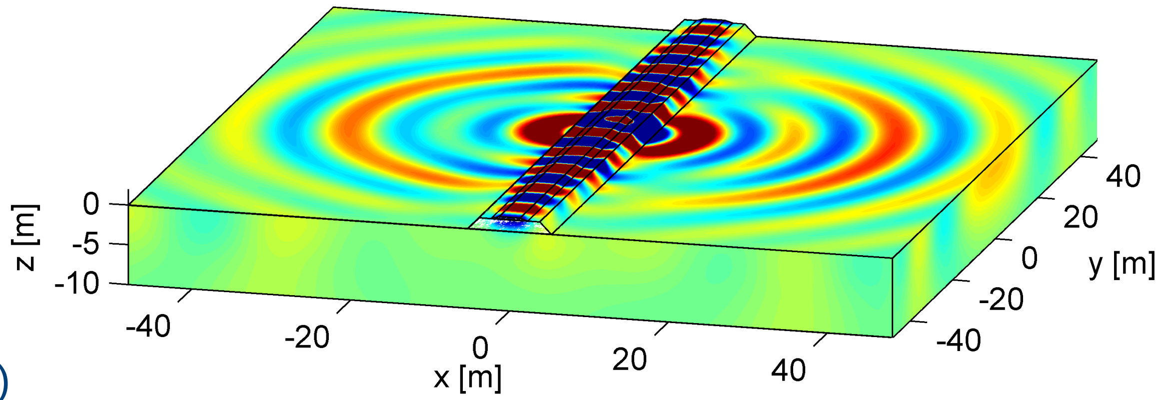
- Geofoam as in-fill material:

C_{s} [m/s]	C_{p} [m/s]	ν [−]	$\beta_{\mathrm{s}} = \beta_{\mathrm{p}}$ [−]	ρ [kg/m^3]
330	495	0.10	0.025	80

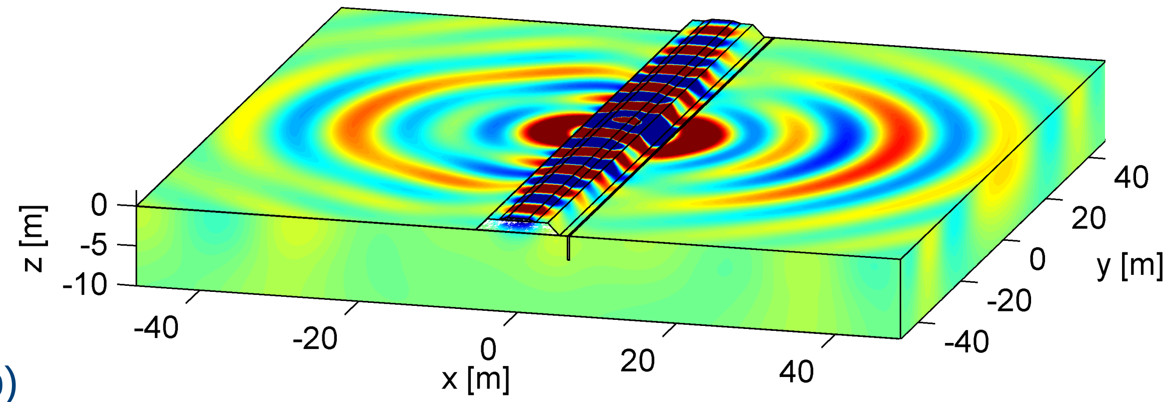
- $f_{\mathrm{c}} = 0.6\frac{C}{h} \simeq 60\,\mathrm{Hz}$

Insertion loss (x, y, z)

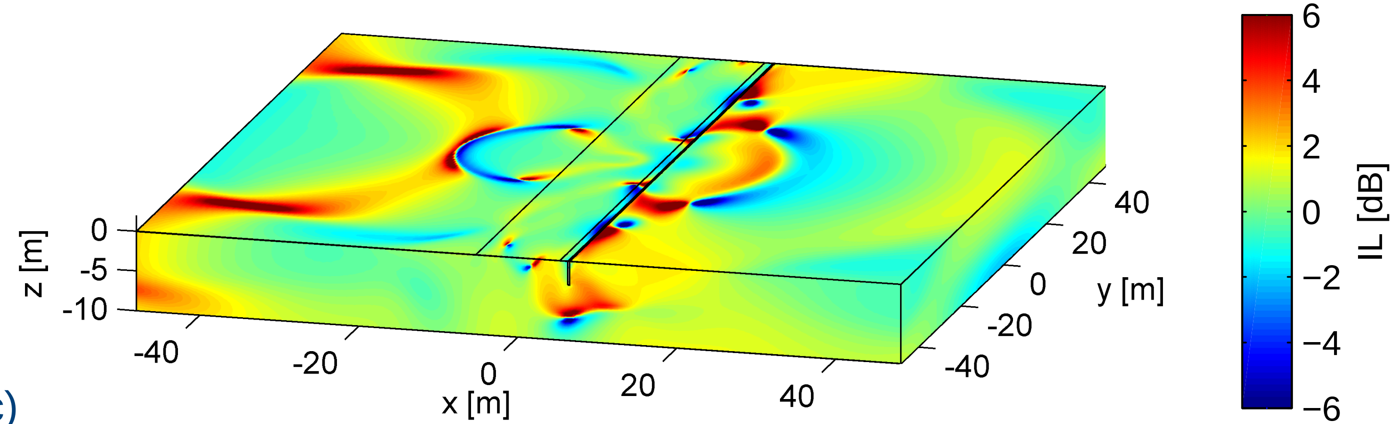
- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of a gefoam filled trench of $3\,\mathrm{m}$ at $30\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).



(a)



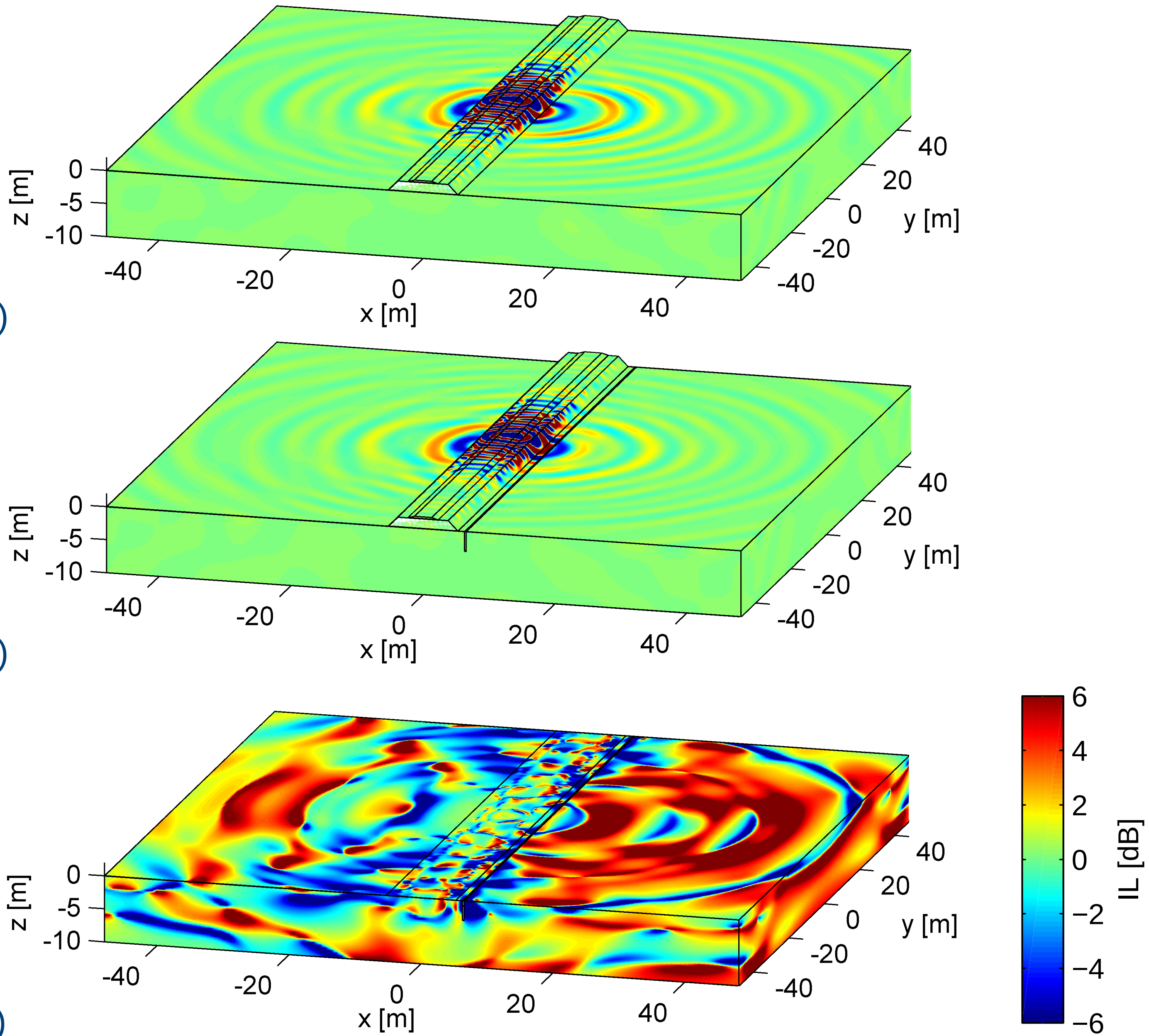
(b)



(c)

Insertion loss (x, y, z)

- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of a gefoam filled trench of $3\,\mathrm{m}$ at $60\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).



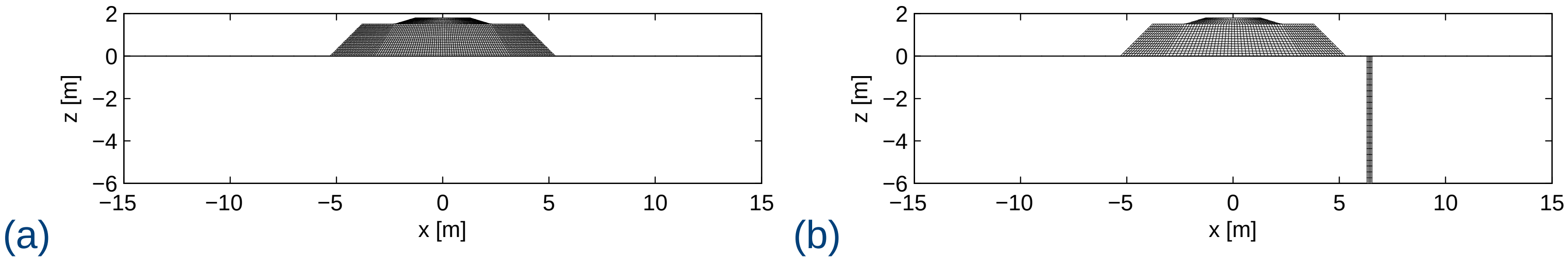
(a)

(b)

(c)

Geofoam filled trench

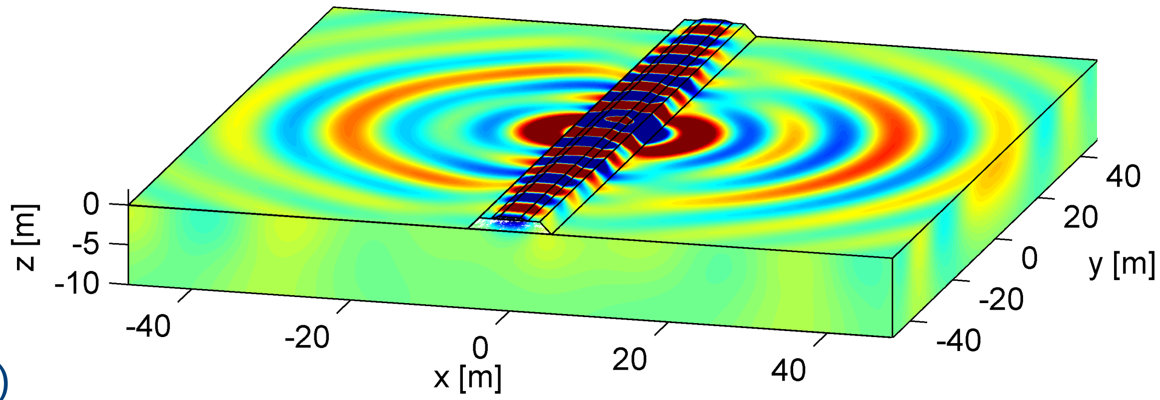
- Rectangular cross section: $w = 0.25\,\mathrm{m}$, $h = 6\,\mathrm{m}$



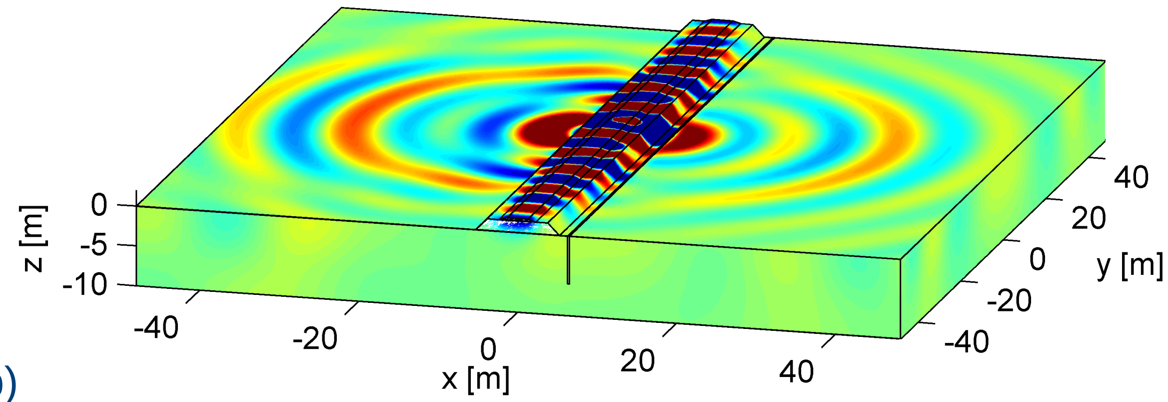
- $f_c = 0.6\frac{C}{h} \simeq 30\,\mathrm{Hz}$

Insertion loss (x, y, z)

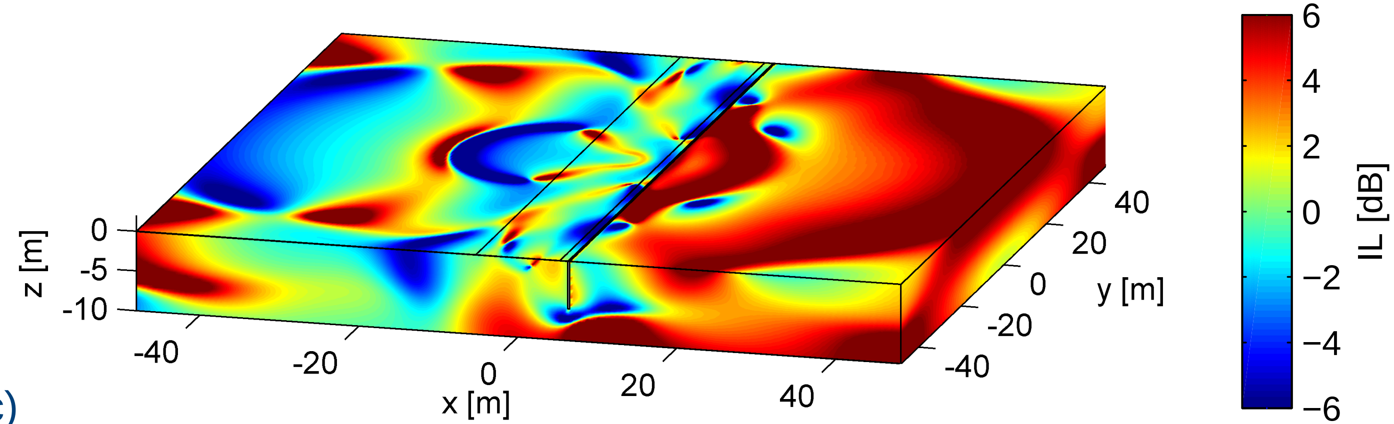
- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of a gefoam filled trench of $6\,\mathrm{m}$ at $30\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).



(a)



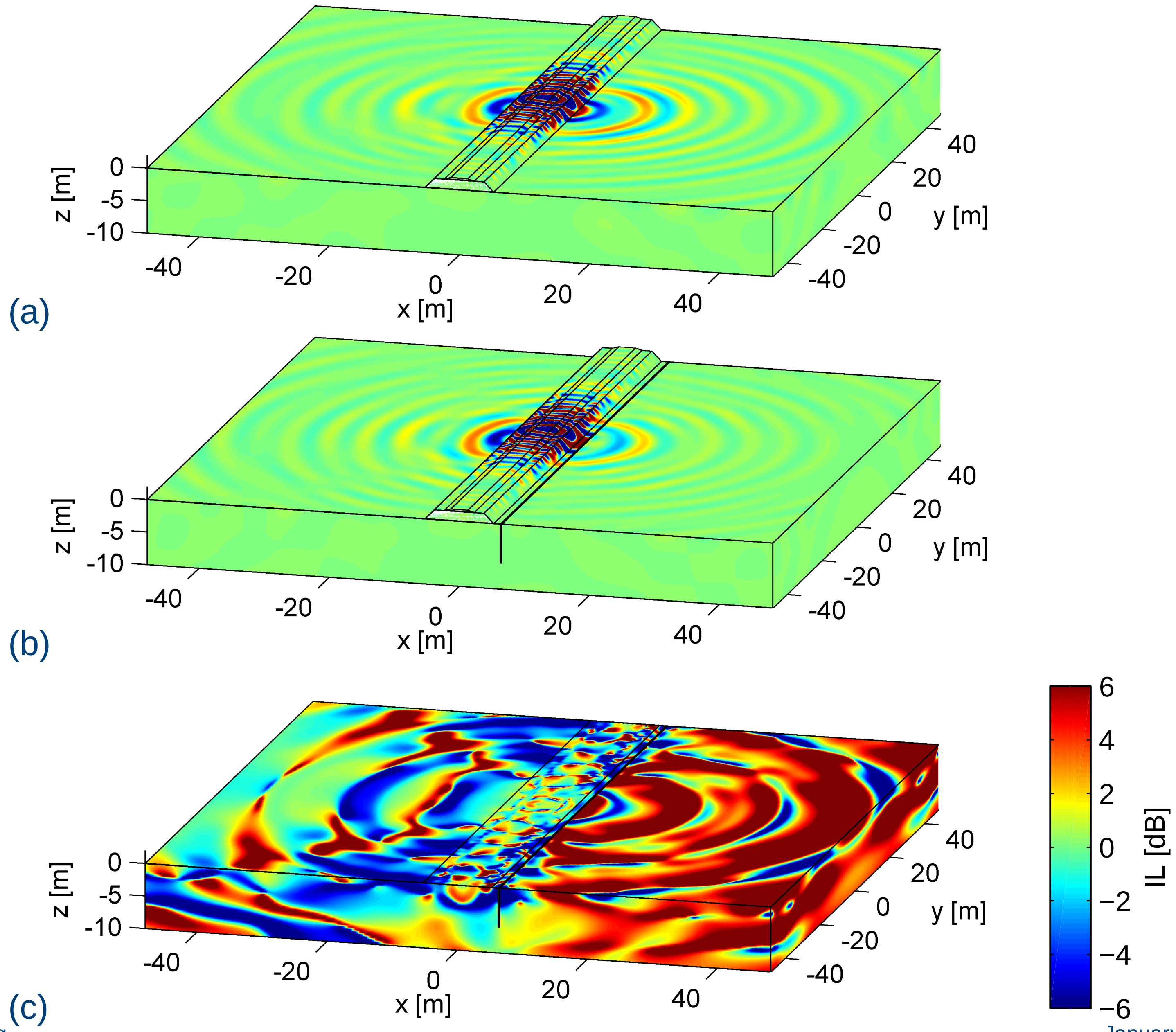
(b)



(c)

Insertion loss (x, y, z)

- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of a gefoam filled trench of $6\,\mathrm{m}$ at $60\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).



Location

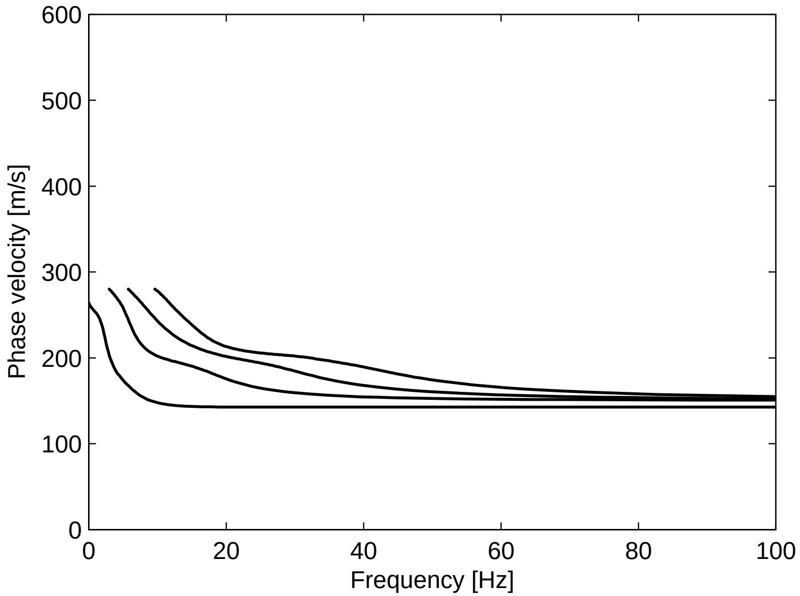
- Site located between Murcia and Orihuela

Horizontally layered visco-elastic halfspace

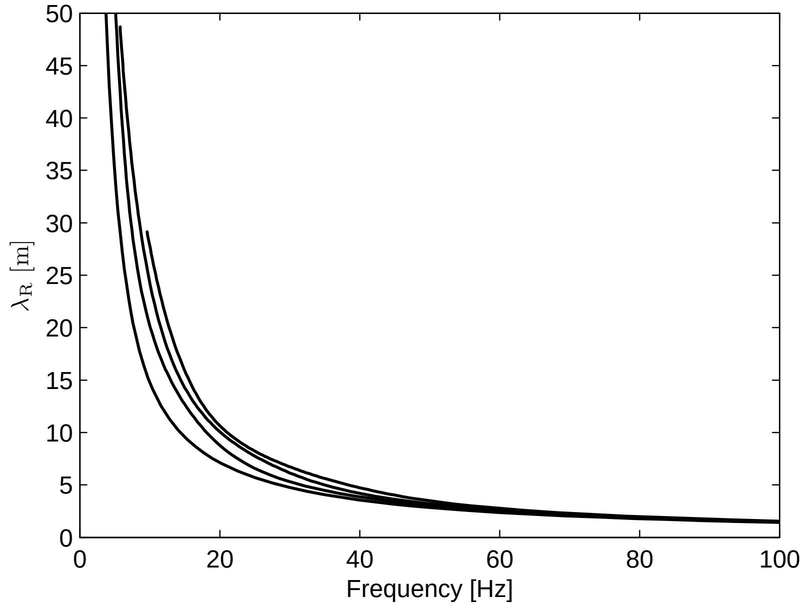
Layer	Thickness [m]	C_{s} [m/s]	C_{p} [m/s]	β_{s} [−]	β_{p} [−]	ρ [kg/m^3]
1	10	150	500	0.025	0.025	1875
2	20	200	665	0.025	0.025	1875
3	10	240	665	0.025	0.025	1885
4	∞	280	690	0.025	0.025	2000

Rayleigh wave dispersion curves

- Dispersive behaviour due to the variation of soil properties with depth
- Multiple modes appear with cut-on frequencies
- (a) Phase velocity C_{R} and (b) wavelength $\lambda_R = C_{\mathrm{R}}/f$ vs. frequency f (first four modes)



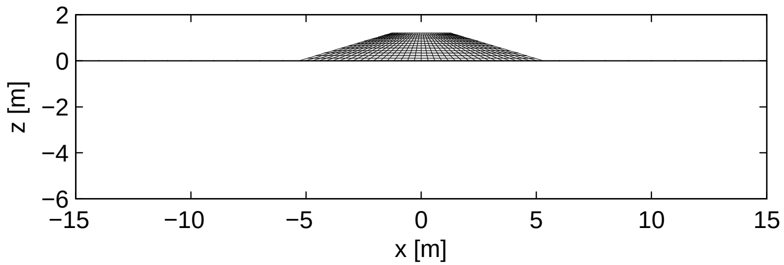
(a)



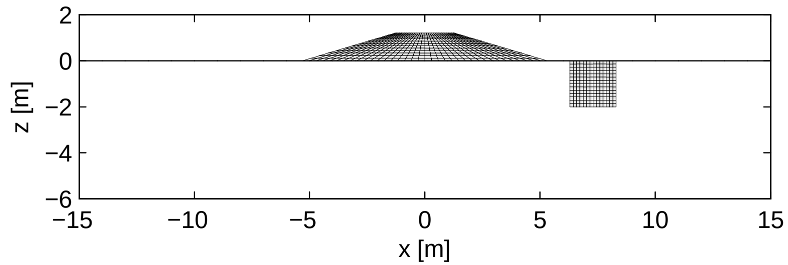
(b)

Variant A

- Rectangular cross section: $w = 2\,\mathrm{m}$, $h = 2\,\mathrm{m}$

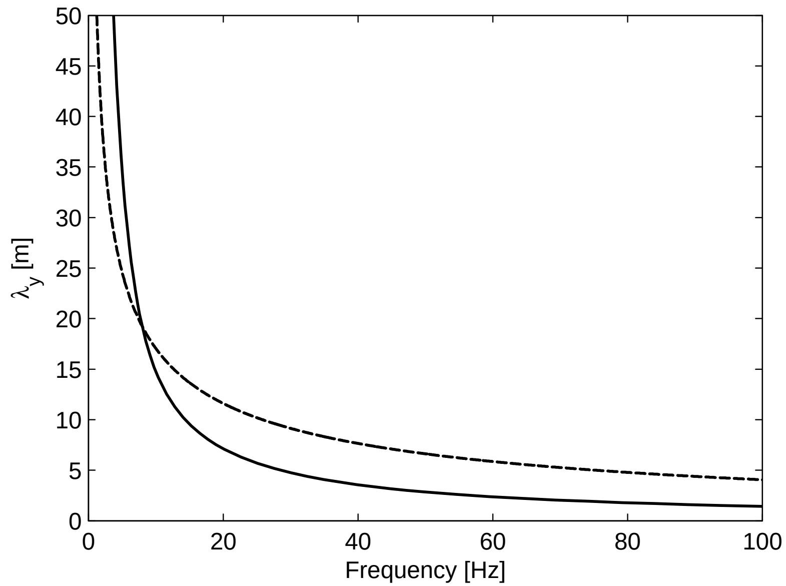


(a)



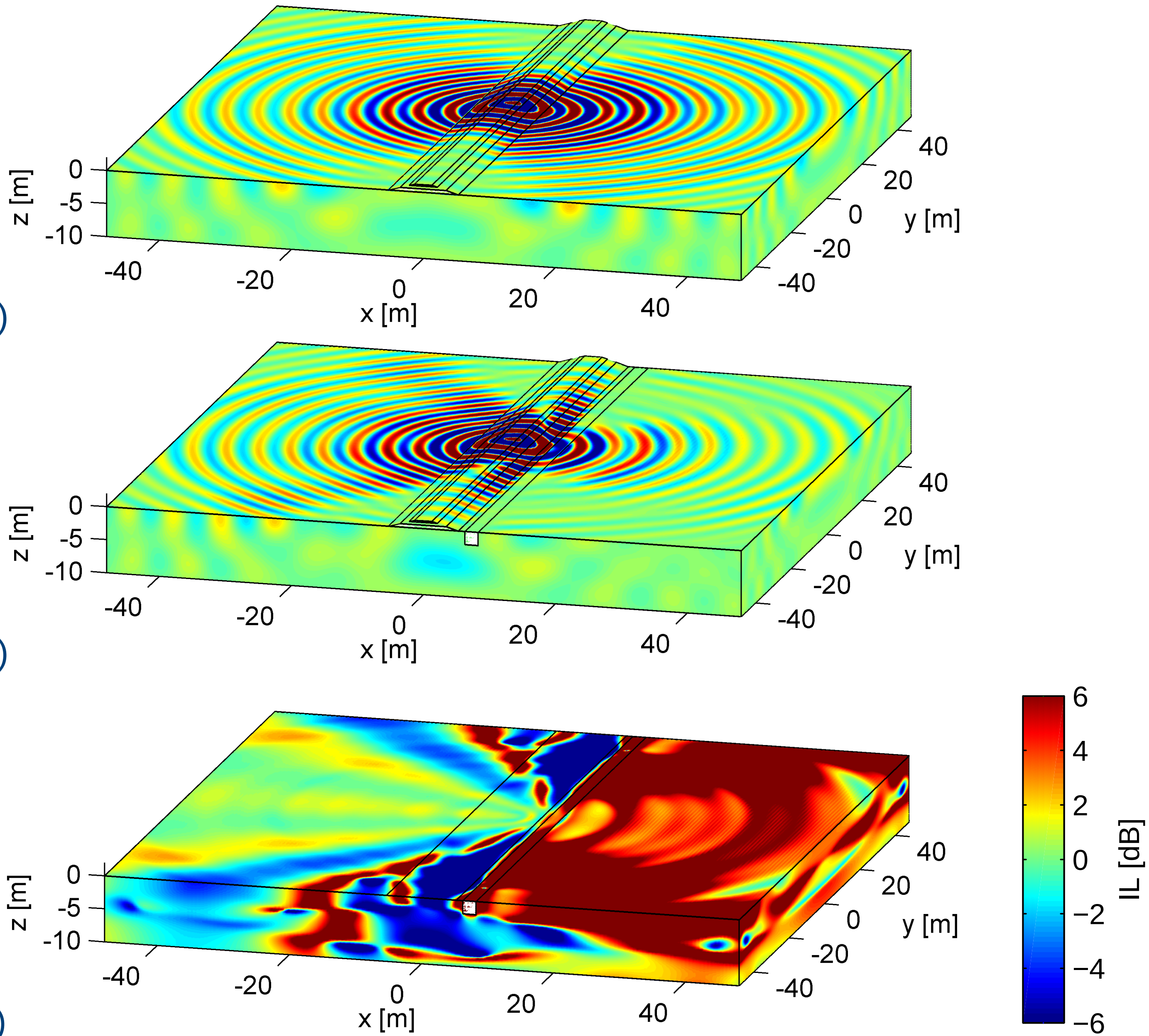
(b)

- Interaction of Rayleigh wave and bending wave:
  - Rayleigh wave dispersion curve (solid line)
  - Free bending wave dispersion curve (dashed line)



Insertion loss (x, y, z)

- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of stiffening next to the track at $30\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).



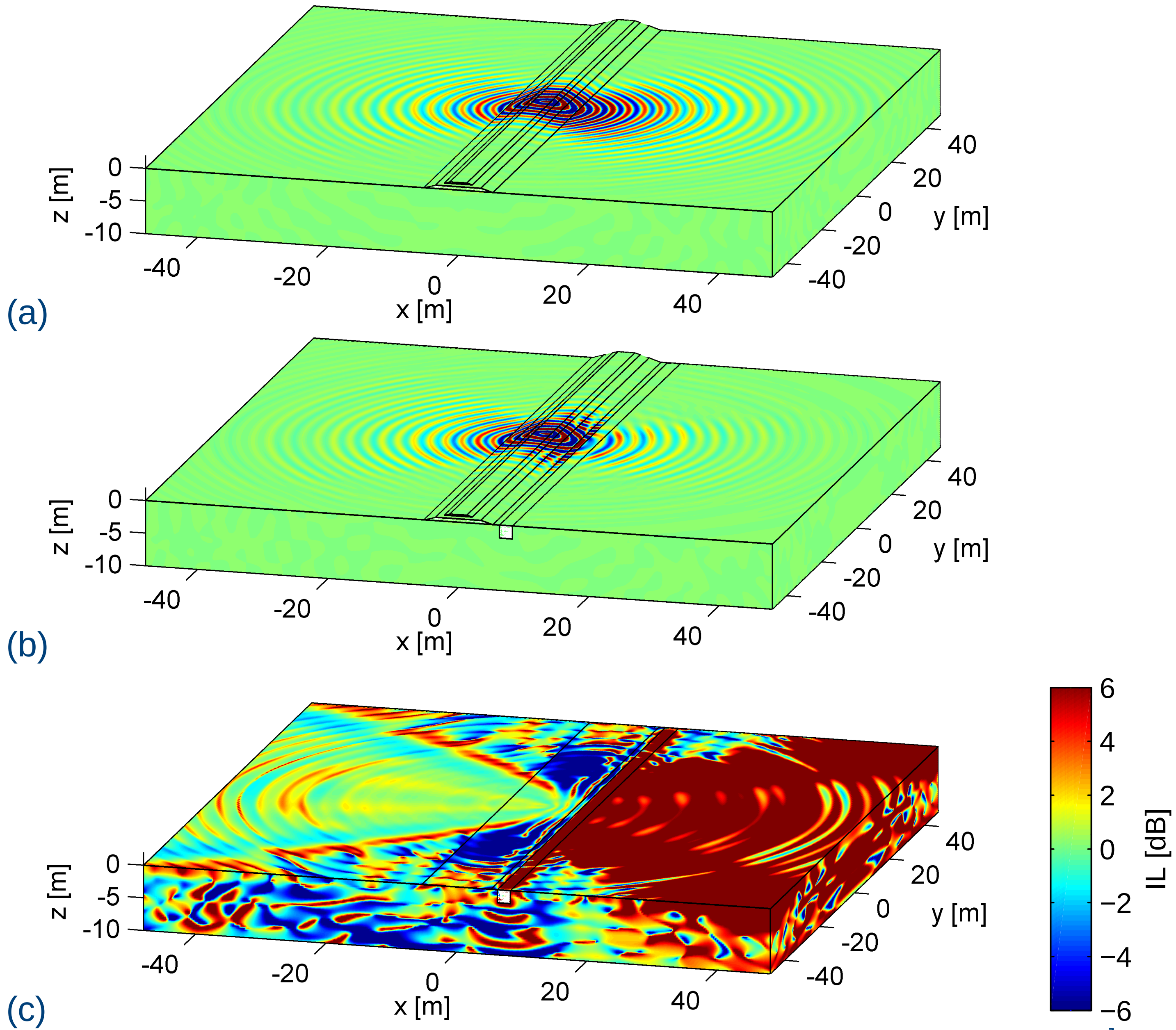
(a)

(b)

(c)

Insertion loss (x, y, z)

- Real part of the vertical displacement field $\hat{u}_z$ in (a) the reference case and (b) in case of stiffening next to the track at $60\,\mathrm{Hz}$. The corresponding insertion loss IL_z is shown in (c).



Physical mechamism

- Subgrade stiffening next to the track: the stiffened block of soil can behave as a wave impeding barrier.
- The wave impeding effect critically depends on the relation between λ_{R} and λ_{b}:
  - Only a reduction of vibration levels possible if $\lambda_{\mathrm{b}} > \lambda_{\mathrm{R}}$ (i.e. if $f > f_{\mathrm{c}}$)
  - Area of significant reduction delimited by $\theta_{\mathrm{c}} = \sin^{-1}\left(\frac{\lambda_{\mathrm{R}}}{\lambda_{\mathrm{b}}}\right)$
  - Increase of depth to be preferred to an increase of width
- An analysis in the (λ_y, ω)–domain provides a useful tool to obtain an efficient design

Gerona test site

- Stiffness contrast between the soil and the stiffened soil is not large
  $\Rightarrow$ only a modest reduction in vibration levels can be obtained

Murcia test site

- Stiffness contrast between the soil and the stiffened soil is large
  $\Rightarrow$ a significant reduction in vibration levels can be obtained